Angular Polyspectra in Cosmology: LSS Bispectrum and CMB Trispectrum

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22 June 2017

Outline

- Polyspectra: Definition and Statistical Meaning;
- Inflation Overview;
- LSS Bispectrum;
- CMB Trispectrum;
- Conclusion.

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Polyspectra: Definition

Given a random field $\Phi(\mathbf{x})$, we define the **n-point correlation function**:

$$\langle \Phi(\mathbf{x}_1) \cdots \Phi(\mathbf{x}_n) \rangle$$

is defined as the excess probability, compared to a random distribution, of finding n points in a certain configuration.

The **polyspectrum** of order n-1 is the Fourier counterpart of the n-point correlation function.

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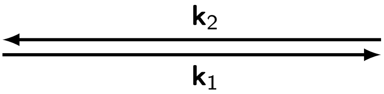
Power Spectrum

Given a field $\Phi(\mathbf{x})$ and its Fourier transform

$$\Phi(\mathbf{k}) = \int d^3x \Phi(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

Power Spectrum:

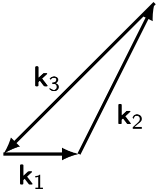
$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) P_\Phi(k_1)$$



Bispectrum

Bispectrum:

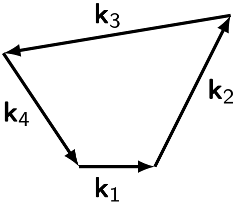
$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\Phi(k_1,k_2,k_3)$$



Trispectrum

Trispectrum:

$$\langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\Phi(\mathbf{k}_4)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\,T_\Phi(k_1,k_2,k_3,k_4)$$



Polyspectra: Statistical definition

In statistics, the polyspectra are related to the momenta of a distribution.

$$P(k) \longrightarrow \text{variance}$$

$$B(k_1, k_2, k_3) \longrightarrow \text{skewness}$$

$$T(k_1, k_2, k_3) \longrightarrow \text{kurtosis}$$

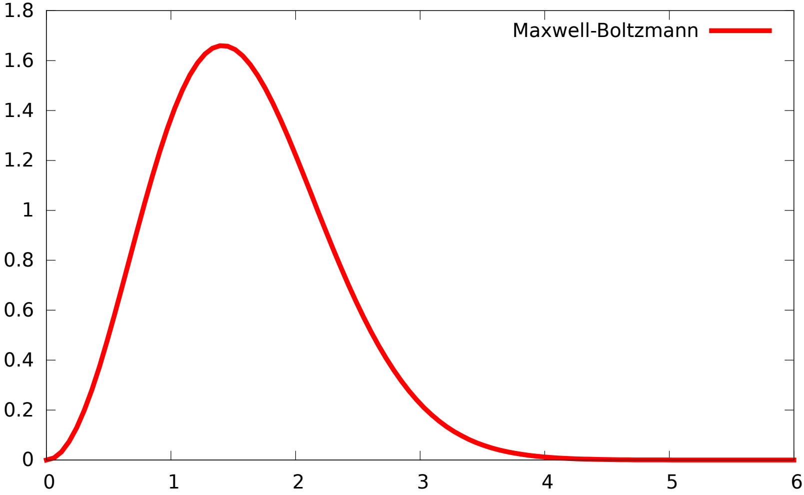
Gaussian distribution

A Gaussian distribution is fully described by the lowest moments: mean (μ) and variance (σ):

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Skewness = Kurtosis = 0

Gaussian vs Non-Gaussian



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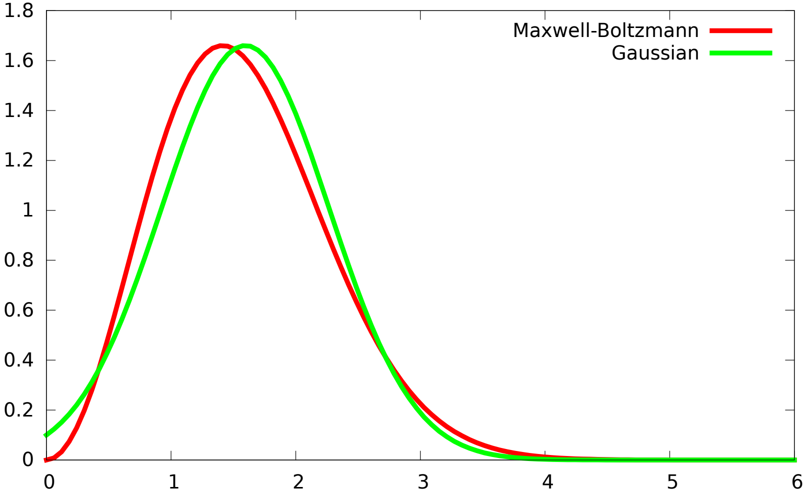
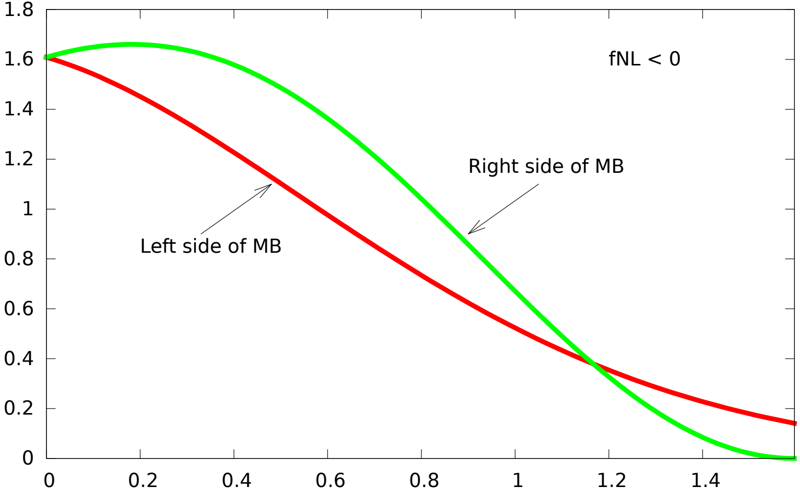


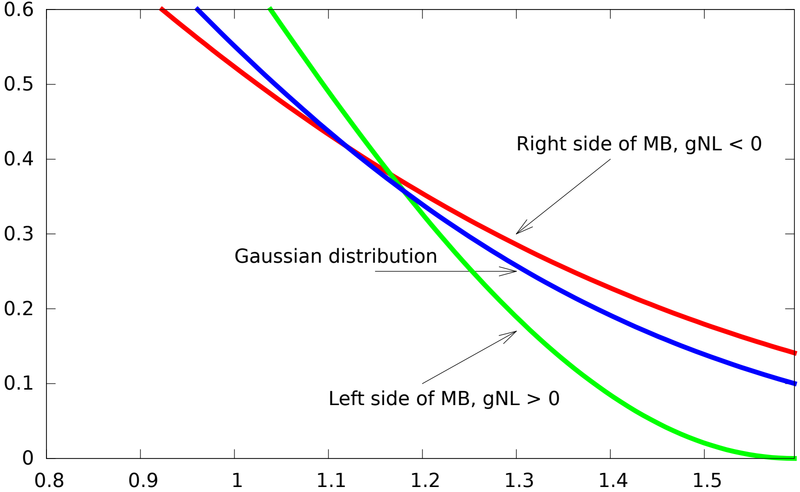
f_{NL}: Skewness

Skewness: asymmetry of the distribution.



g_{NL}: Kurtosis

Kurtosis: height of the tails of the distribution.



Polyspectra on the Sphere

If the field is defined on the sphere, the Fourier transform is performed using the **Spherical Harmonics**, i.e. the basis for the functions defined on S^2.

$$\Phi(\mathbf{x}) = \int d^3\mathbf{x}\Phi(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}} \longrightarrow \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm}Y_{lm}(\mathbf{x})$$

$$\Phi(\mathbf{k}) \longrightarrow a_{lm}$$

$$P(k) \longrightarrow C_l$$

$$B(k_1, k_2, k_3) \longrightarrow B_{l_1 l_2 l_3}$$

$$T(k_1, k_2, k_3, k_4) \longrightarrow T_{l_1 l_2 l_3 l_4}$$

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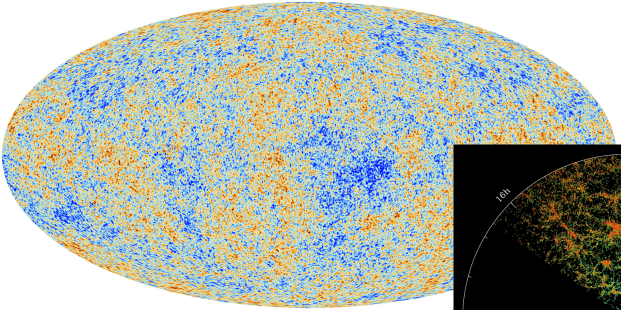
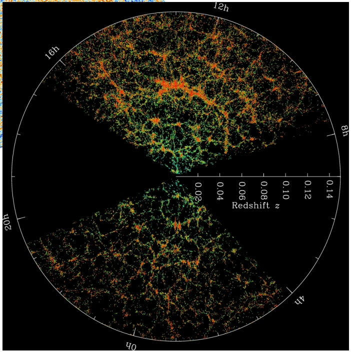
What do we study?



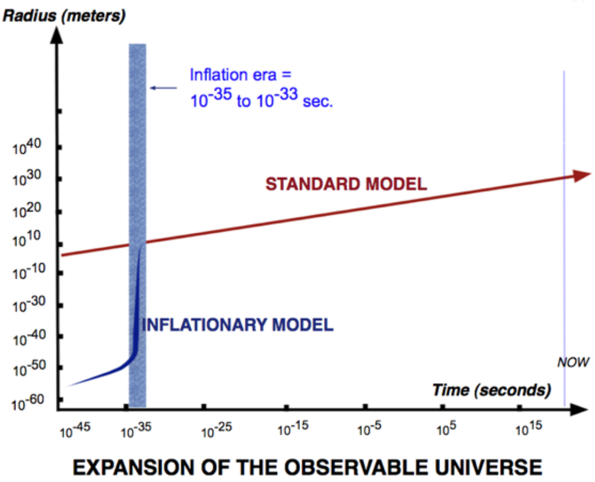


What do they have in common?

Inflationary seeds

Both CMB and LSS were seeded by Inflaton vacuum perturbation!

Inflation


Radius (meters)
Inflation era =
10^{-35} to 10^{-33} sec.
10^{40}
10^{30}
10^{20}
10^{10}
10^{-10}
10^{-30}
10^{-40}
10^{-50}
10^{-60}
STANDARD MODEL
INFLATIONARY MODEL
NOW
Time (seconds)
10^{-45}
10^{-35}
10^{-25}
10^{-15}
10^{-5}
10^{5}
10^{15}
EXPANSION OF THE OBSERVABLE UNIVERSE

Inflaton and Inflation

INFLATON $\phi(\mathbf{x}, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$

- Spatial average of the field → $\phi_0(t)$
- Vacuum Fluctuations → $\delta\phi(\mathbf{x}, t)$

Inflation

After Inflation

Vacuum Fluctuations → Primordial Gravitational Fluctuations → Matter Fluctuations → CMB Anisotropy

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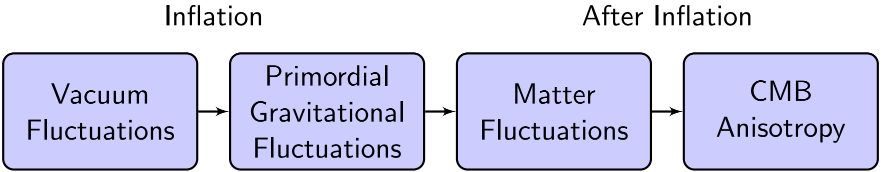

Primordial Gravitational Field

We can write the primordial gravitational field $\Phi(\mathbf{x})$ by mean of the so-called Bardeen potential:

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}(\Phi_L(\mathbf{x})^2 - \langle \Phi_L(\mathbf{x})^2 \rangle) + g_{NL}[\Phi_L(\mathbf{x})^3]$$

Gaussian component

non-Gaussian component

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Gaussian component

non-Gaussian component

Non-Gaussianity

The amplitude of f_{NL} and g_{NL} affect both the photon and the matter distribution.

CMB: the distribution of the anisotropies inherits the primordial non-gaussianity.

LSS: f_{NL} and g_{NL} affect the way the gravitational collapse happens.

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Last Remark

There is NOT only one Inflation model.

Single-Field

- Standard Scenario
- k-inflation
- DBI inflation
- ...

Multi-Fields

- Curvaton Scenario
- Ghost inflation
- D-cceleration scenario
- ...

Which is the correct one?

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Last Remark

Standard Model

The primordial fluctuation distribution is GAUSSIAN.

Non-Standard Models

The primordial fluctuation distribution is NON-GAUSSIAN.

The amounts of non-Gaussianity depends on the model.

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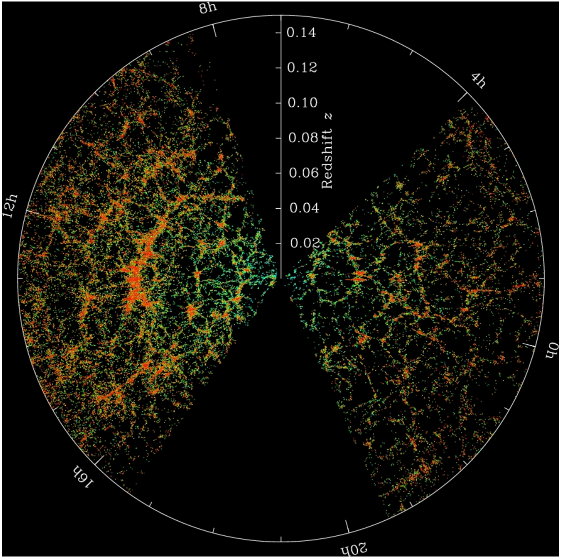
Large Scale Structure of the Universe



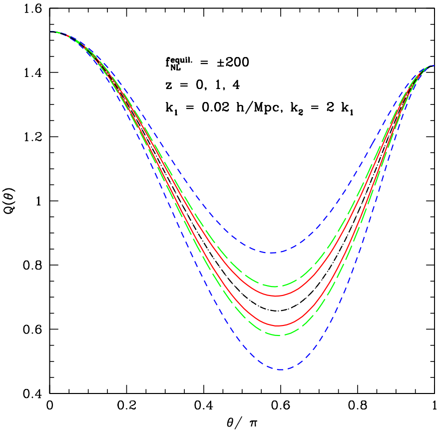
Matter Bispectrum

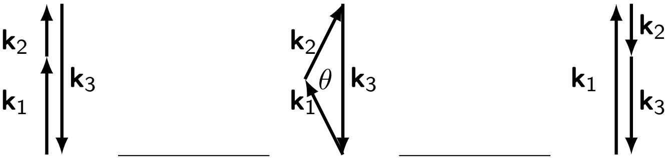
LSS bispectrum is composed by two terms:

$$B(k_1, k_2, k_3) = B_I(k_1, k_2, k_3) + B_G(k_1, k_2, k_3),$$

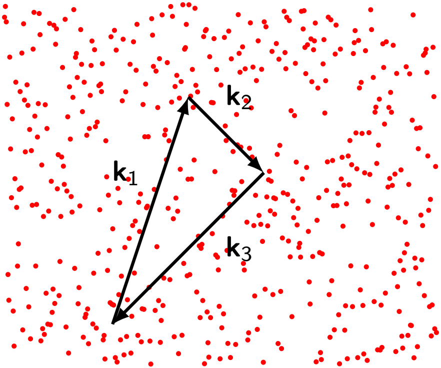
where B_I is the primordial bispectrum, parametrized by f_{NL}, and B_G is the gravitational evolution bispectrum.

Matter Bispectrum

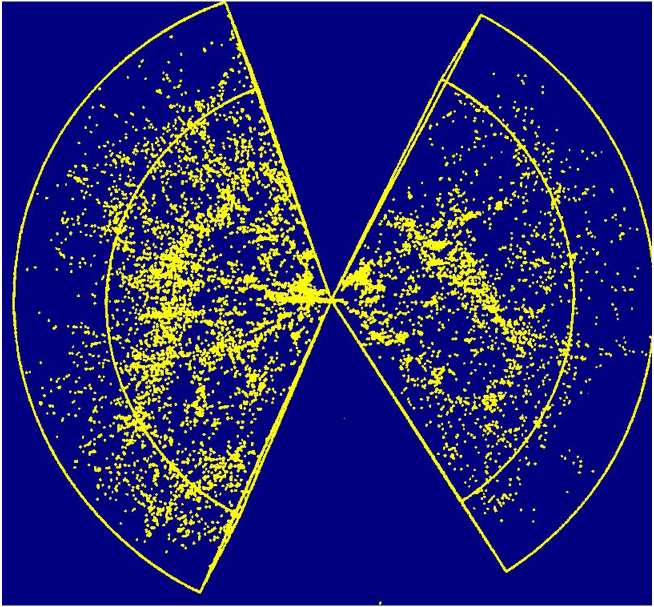




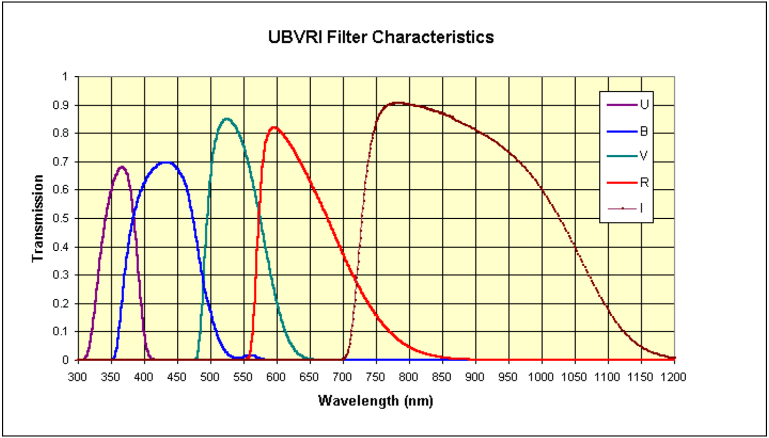
3D triangle configuration



Photometric Samples



Photometric Filters

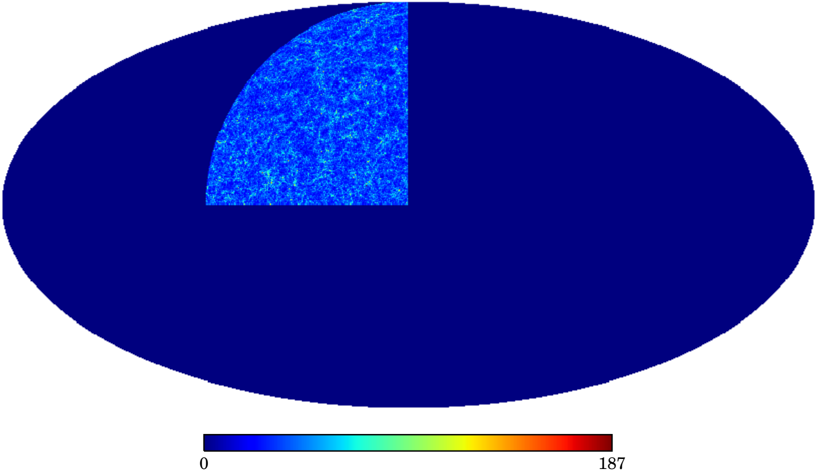


Photometric Error

$$\phi(z) = \frac{dN_g}{dz} \int dz_p \, P(z|z_p) W(z_p)$$

Dataset

Measurements performed on 125 LSS mocks covering 1/8 of sky at $z = 0.5$ from MICE catalogue (Fosalba et al. 2008, Crocce et al. 2010).



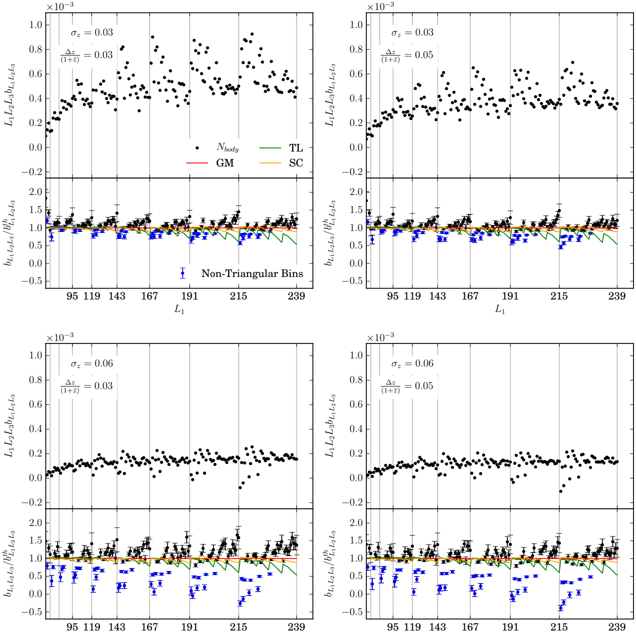
Number of galaxies

Results

small binsize | large binsize

Small photo-z error

Large photo-z error



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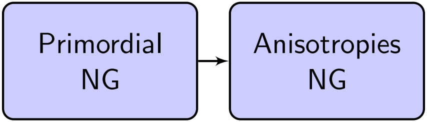
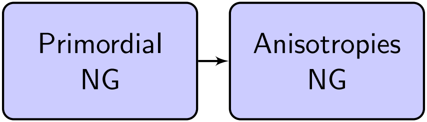
CMB Non-Gaussianity



Evaluate NG in the CMB allows to constrain the inflationary models.

CMB Non-Gaussianity



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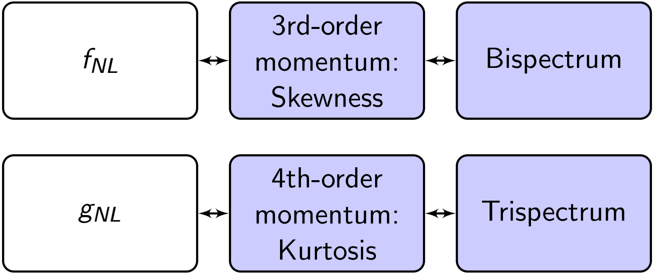
NG Estimators



State of the Art

Optimal Estimator : Unbiased estimator with the lowest variance among the other ones. Planck Collaboration (2015):

$$f_{NL} = 2.5 \pm 5.7 \quad (1\sigma)$$

Single-field models are preferred.

State of the Art

Planck Collaboration (2015):

$$g_{NL} = (-9.0 \pm 7.7) \times 10^4 \quad (1\sigma)$$

g_{NL} has a very low statistical significance!

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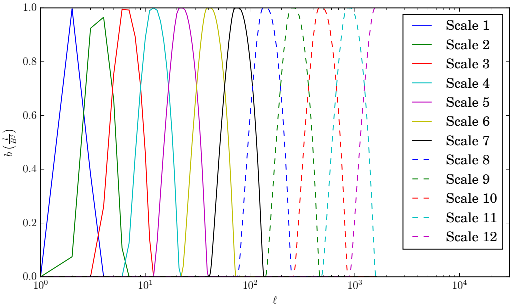
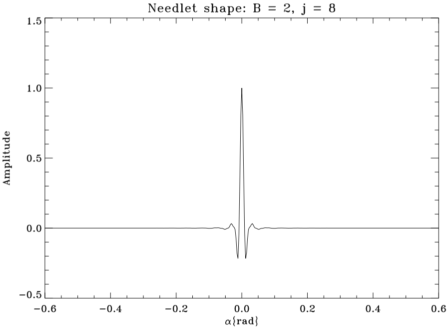
Spherical Needlet System

$$\psi_{jk}(x) := \sqrt{\lambda_{jk}} \sum_{l} b\left(\frac{l}{B^j}\right) \sum_{m=-l}^{l} Y_{lm}(\xi_{jk}) \overline{Y}_{lm}(x)$$

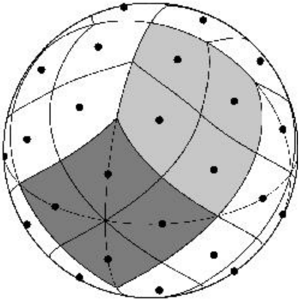


Localization Property



Localization property in real space means raise of the statistical significance in presence of incomplete sky coverage (i.e. ALWAYS).

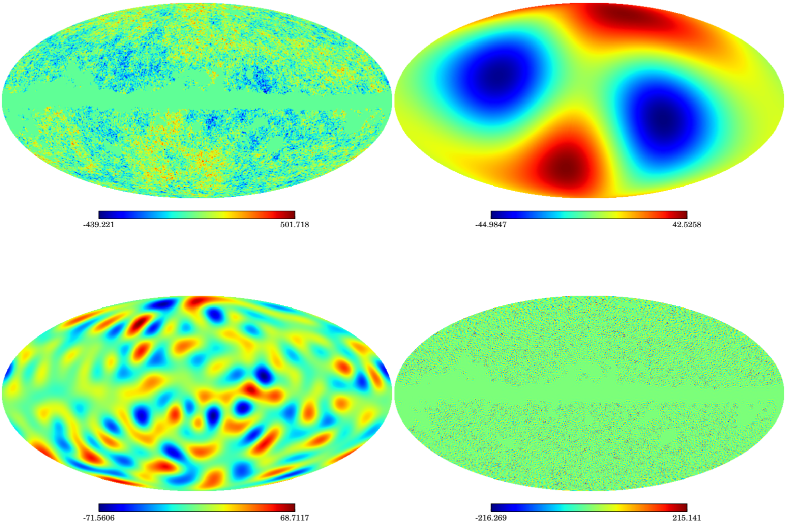
Needlet Coefficients



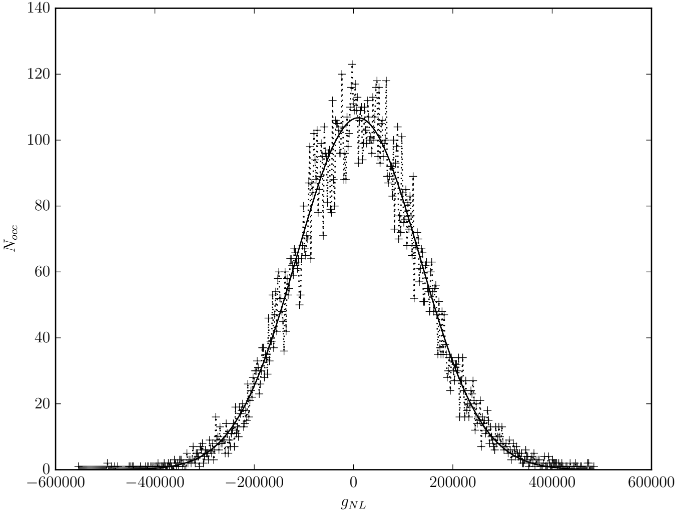
Reconstruction Formula:

$$T(x) = \sum_{l,m} a_{lm} Y_{lm}(x) = \sum_{j,k} \beta_{jk} \psi_{jk}(x)$$

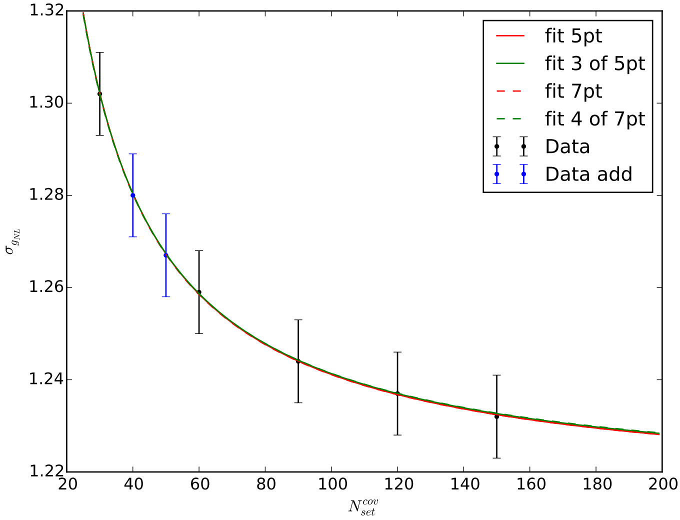
Needlet Deconvolution



g_{NL} Estimation



$\sigma_{g_{NL}}$ Estimation



Future Perspectives

- The angular bispectrum of LSS seems to work well when compared with predictions, now it is time to constrain parameters with it;
- The Needlet trispectrum of CMB is computational feasible and ready to constrain g_{NL} from PLANCK data.