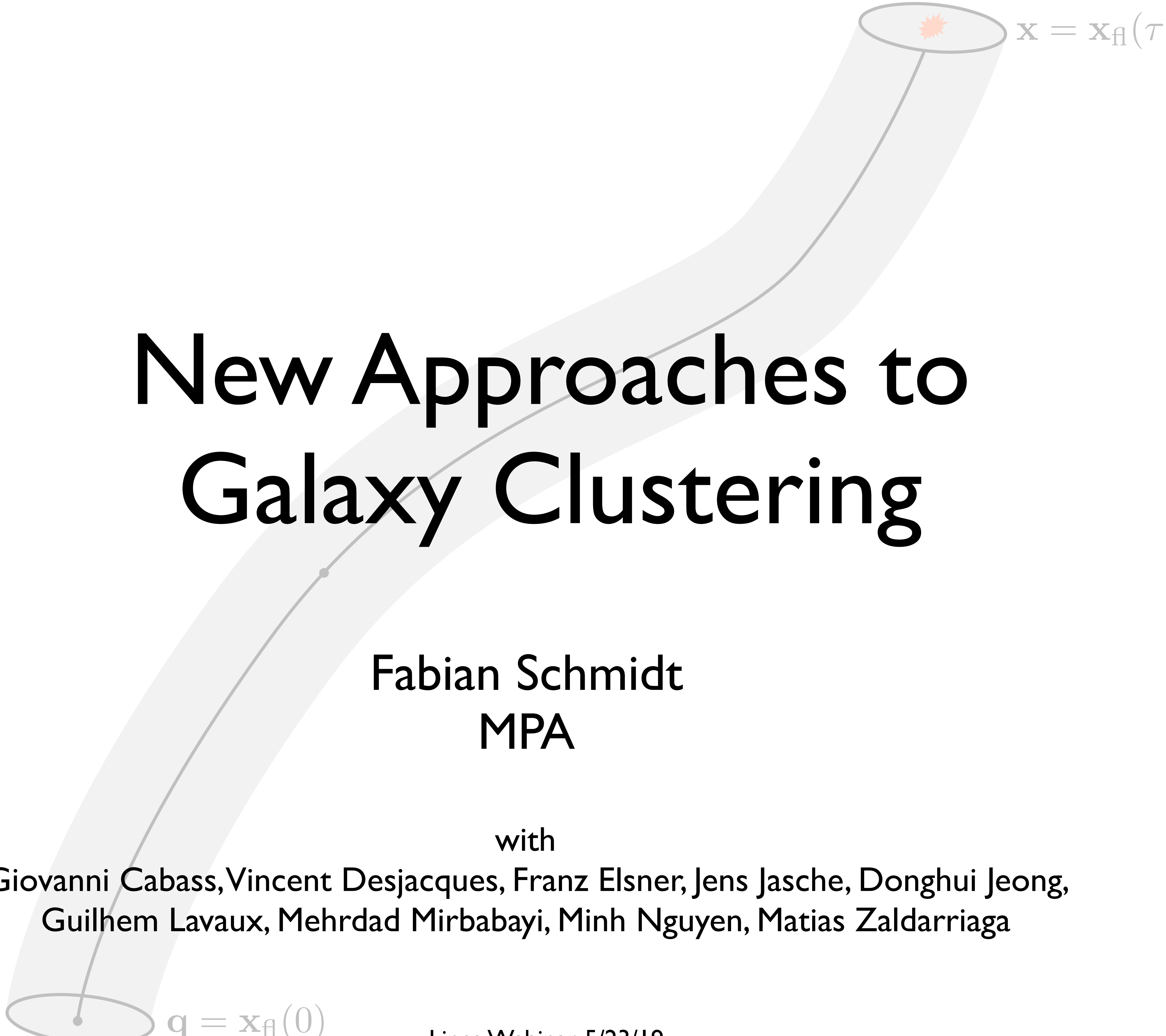
New Approaches to Galaxy Clustering

Fabian Schmidt
MPA

with
Giovanni Cabass, Vincent Desjacques, Franz Elsner, Jens Jasche, Donghui Jeong, Guilhem Lavaux, Mehrdad Mirbabayi, Minh Nguyen, Matias Zaldarriaga



Linea Webinar 5/23/19

Motivation

- The clustering of galaxies (large-scale structure, LSS) is historically one of the key probes of cosmology

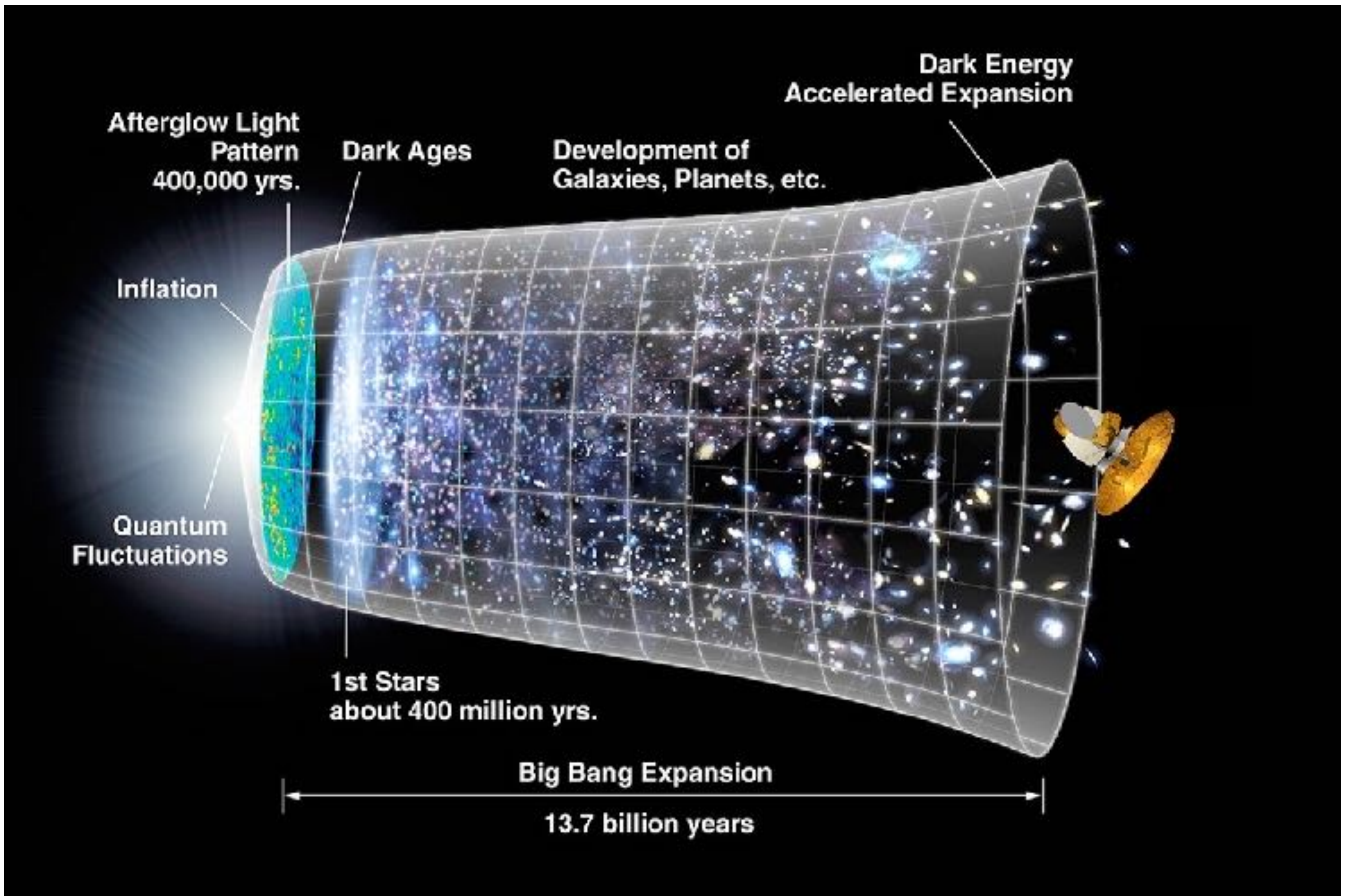
  Peebles; Efstathiou+ '90 predicted a positive cosmological constant Λ from LSS observations

- From ~1998 until recently, most spectacular results came from "cleaner" probes - Supernovae and the cosmic microwave background (CMB)

- Now, again, in a new golden age of LSS with plenty of experiments under way: BOSS, DES, DESI, PFS, SphereX, Euclid, WFIRST, ...

Motivation

- Using large-scale structure, we can learn about



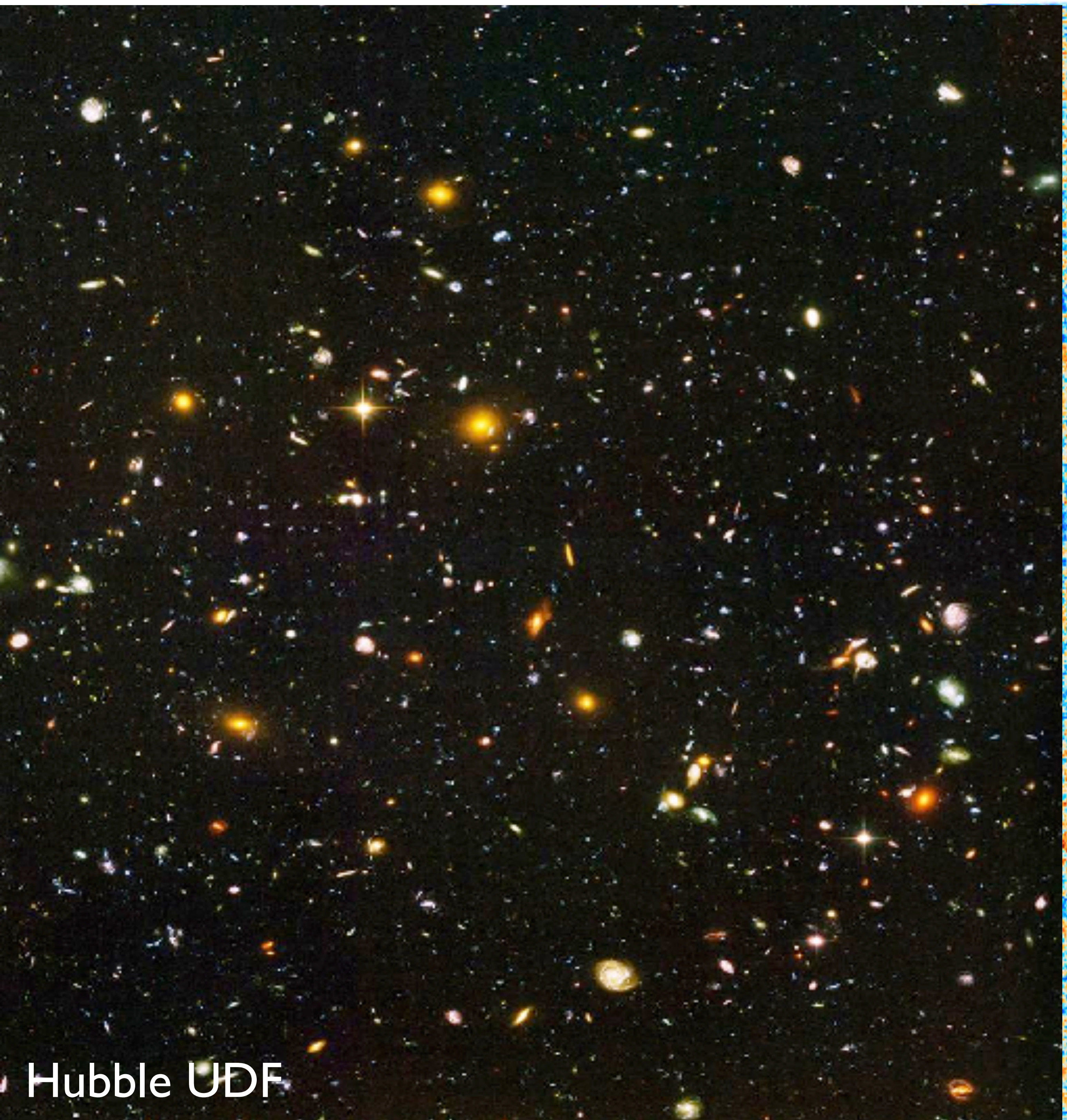
Motivation

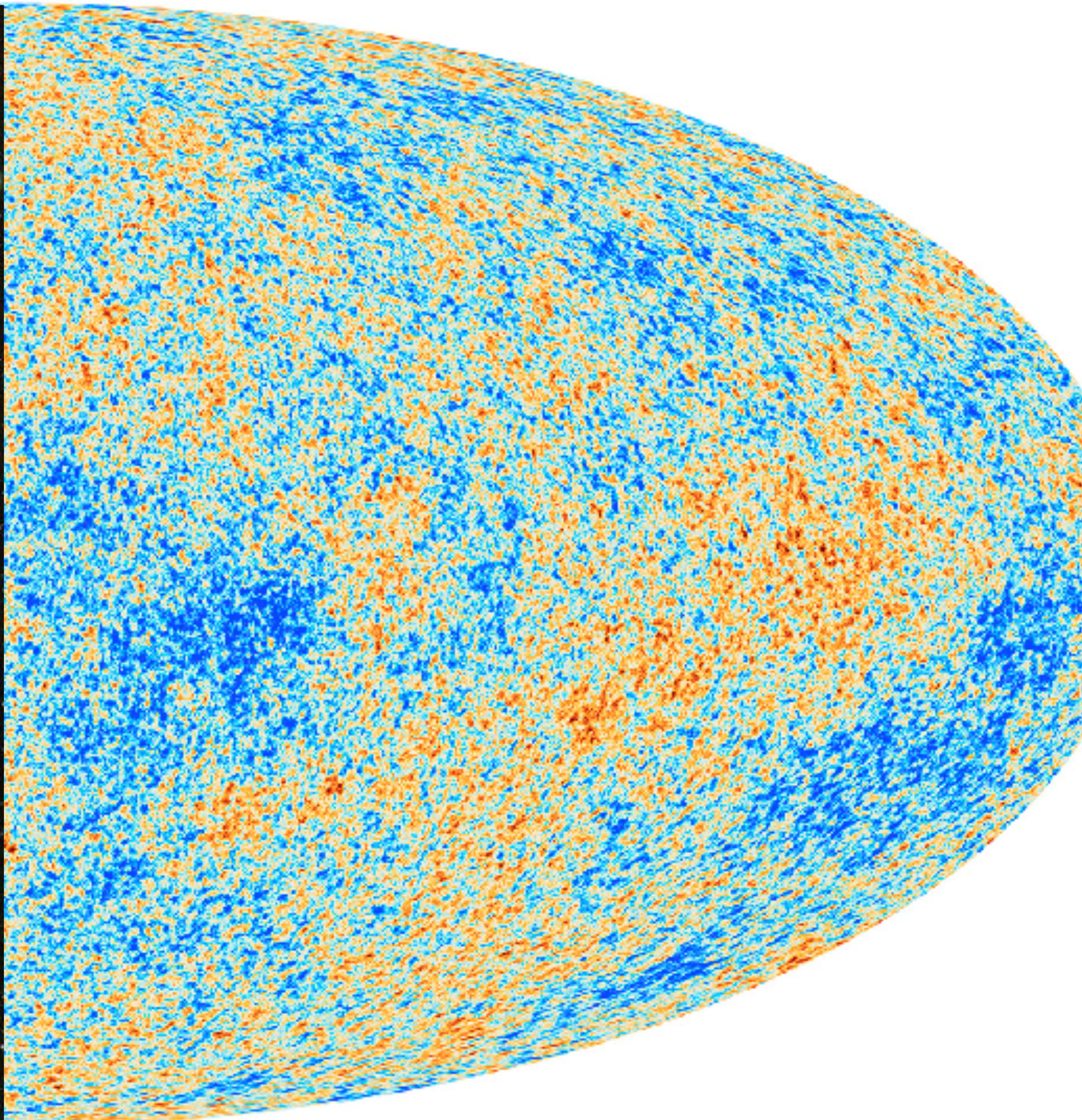
- Inflation: reconstruct the properties of the initial conditions, and look for gravitational waves
- Dark Energy and Gravity: the growth of structure depends sensitively on the expansion history of the Universe, and the nature of gravity

  Growth equation: $D'' + aH\,D' = 4\pi\,G\,\bar{\rho}\,D$

- Dark Matter: how "cold" is cold dark matter ? What is the sum of neutrino masses ?

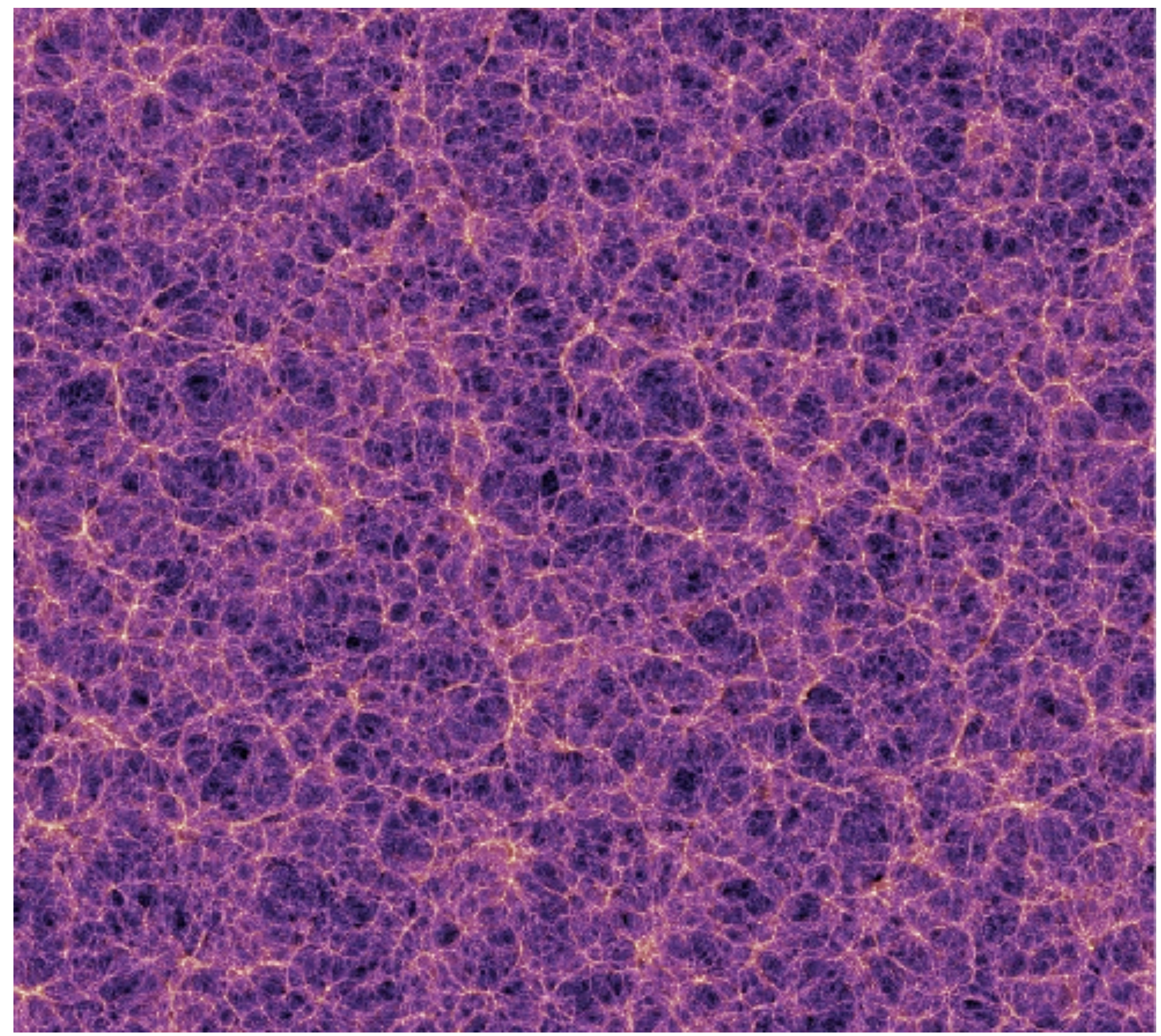
Challenge: unlike the CMB, every data point is nonlinear!





Cold Dark Matter cosmology in a nutshell

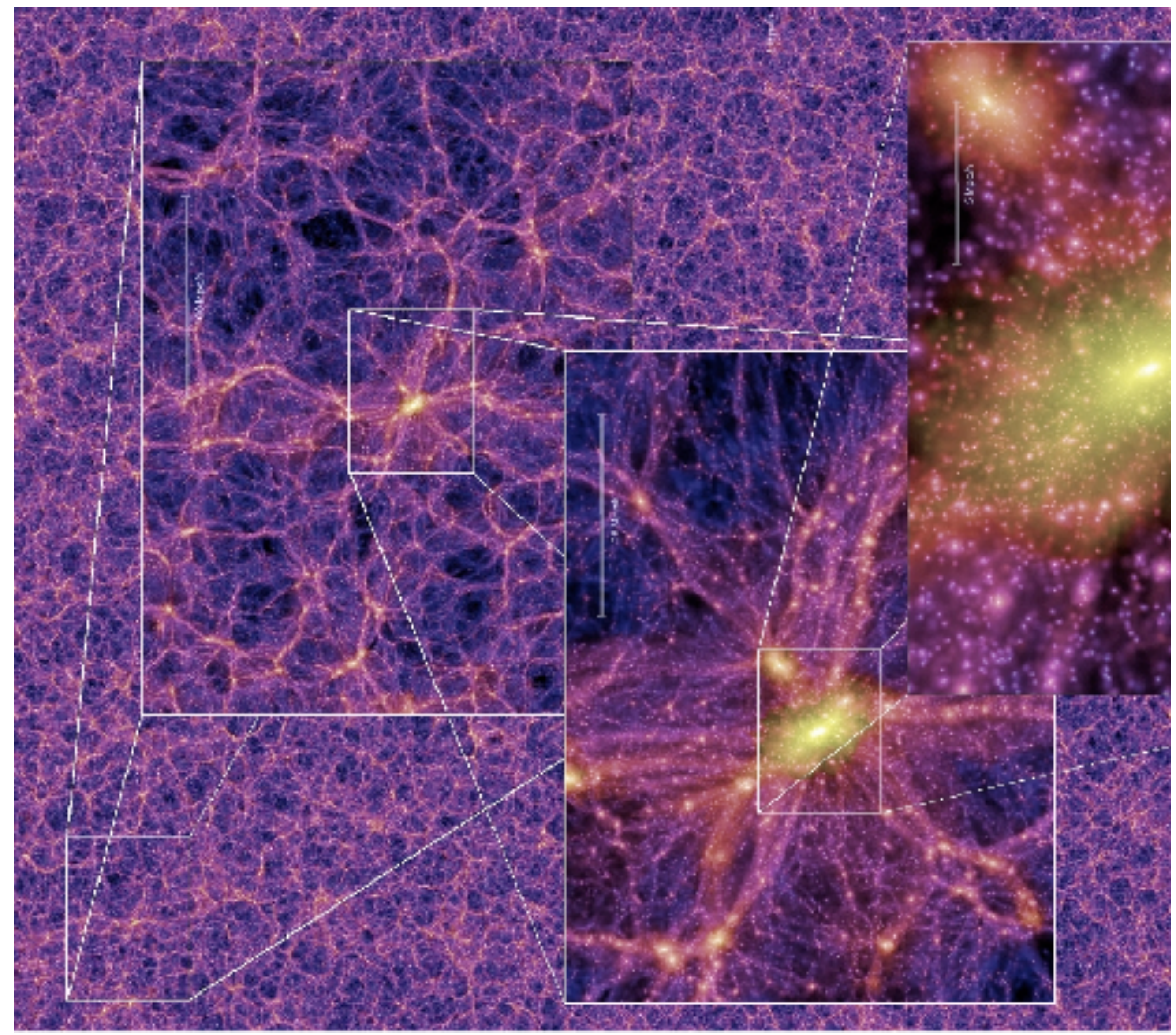
- Assume scale-invariant, adiabatic, approx. Gaussian initial conditions
- Large-scale fluctuations are small (still linear today)
- Structure forms *hierarchically from small to large* scales
- *Perturbative expansion* in fluctuations on large scales



Millennium simulation / MPA

Cold Dark Matter cosmology in a nutshell

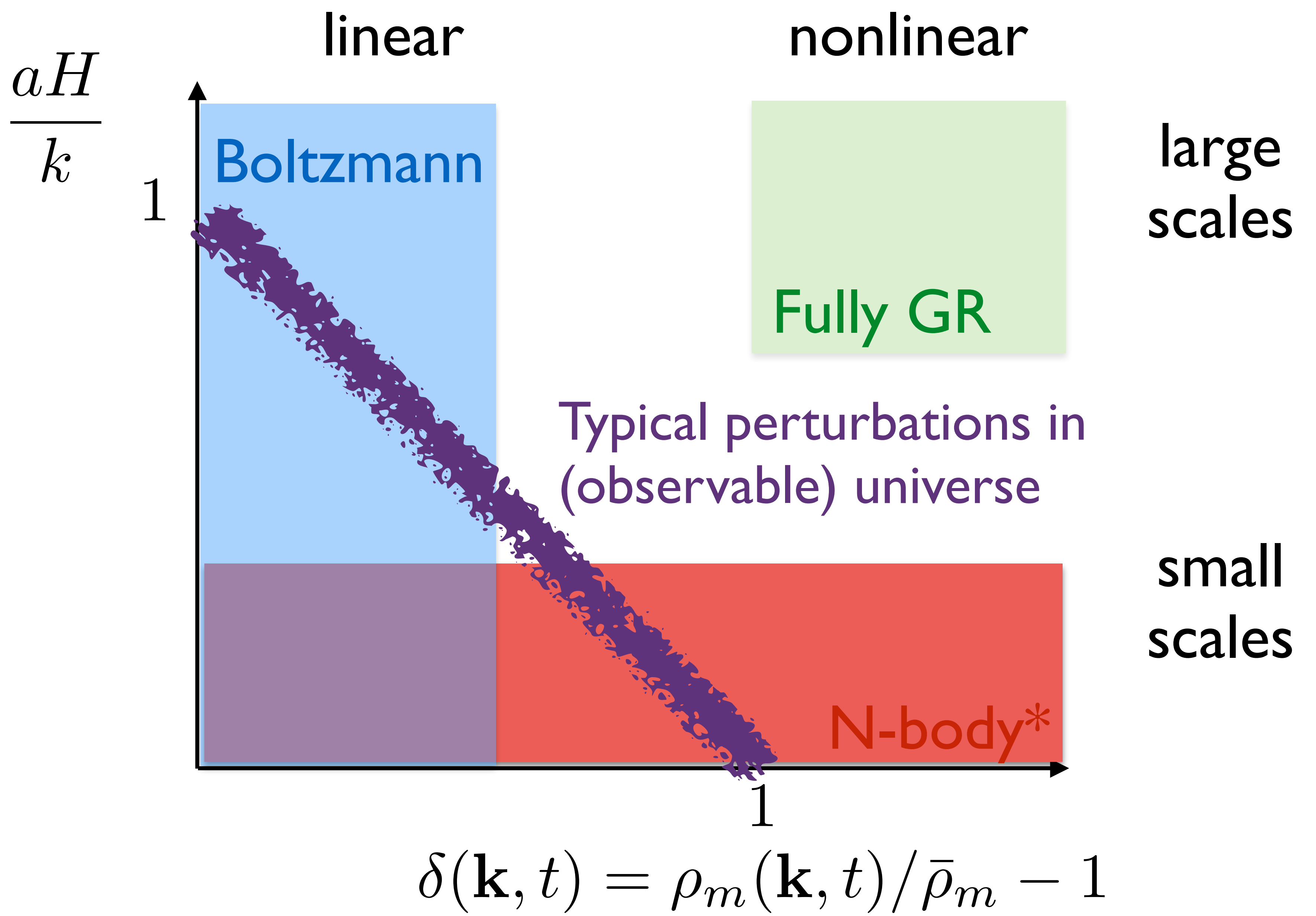
- Assume scale-invariant, adiabatic, approx. Gaussian initial conditions
- Large-scale fluctuations are small (still linear today)
- Structure forms *hierarchically from small to large* scales
- *Perturbative expansion* in fluctuations on large scales



Millennium simulation / MPA

Theory of Large-Scale Structure

- Well-established tools:
  - linear Boltzmann
  - N-body* methods



* and hydrodynamics

Theory of Large-Scale Structure

- Foundation: separation between nonlinear scale and horizon

$$k_{\rm NL} \simeq 0.1 h\,{\rm Mpc}^{-1} \gg aH$$

- Linear theory: Fourier modes evolve independently; *solved problem*
- However, bulk of information in LSS is on nonlinear scales ($N_{modes} \sim k_{max}^3$)

How do we compare theory with data?

- Assume we observe the matter density field* $\rho(\boldsymbol{x}) = \bar{\rho}[1 + \delta(\boldsymbol{x})]$
- Given cosmological model, theory predicts

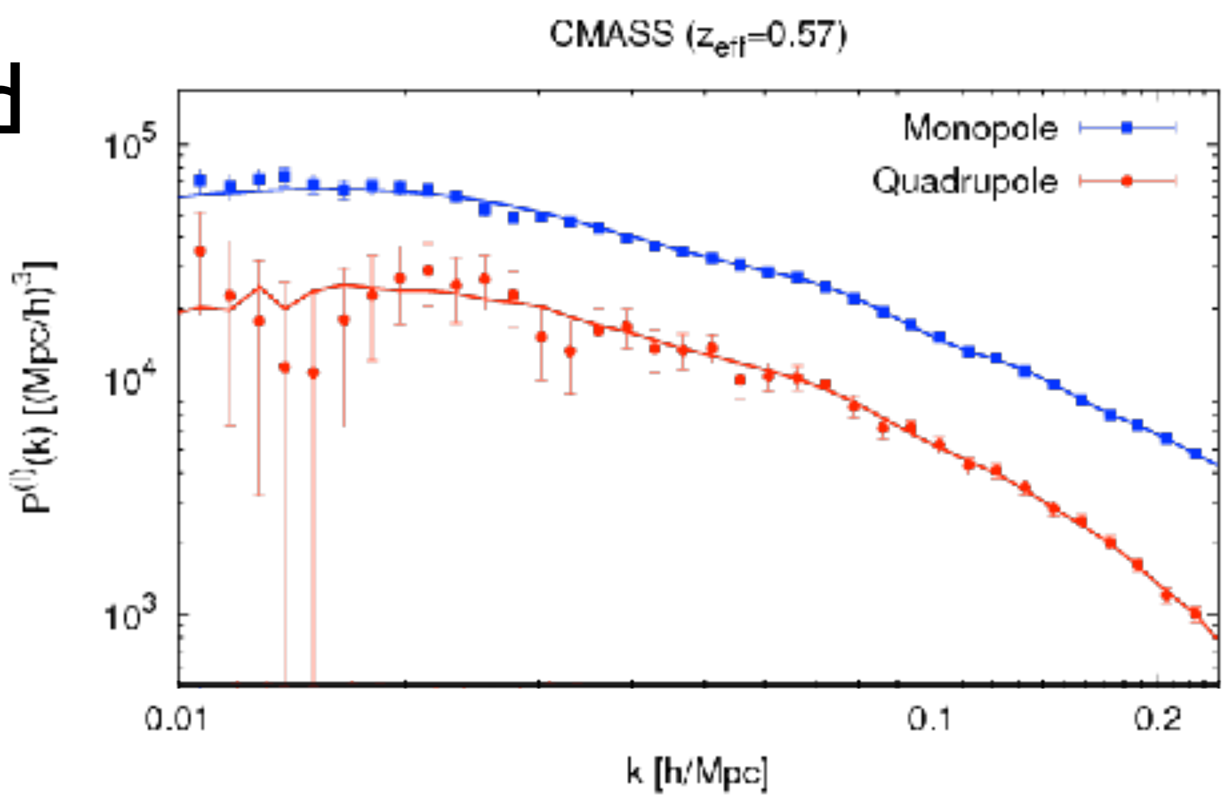
1. Statistics of initial conditions $\delta_{\rm in}(\boldsymbol{x}) = \lim_{t \to 0^+} \delta(\boldsymbol{x}, t)$
2. How a given $\delta_{\rm in}(\boldsymbol{x})$ evolves into the final density field

* Drop time argument throughout for clarity; assume fixed observation time

How do we compare theory with data?

- If matter density field was Gaussian,
  - PDF of δ(x) is multivariate Gaussian, with diagonal covariance in Fourier space
- Then all the information would be contained in the power spectrum

$$\langle \delta(\boldsymbol{k}) \delta^*(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}')\, P(k)$$

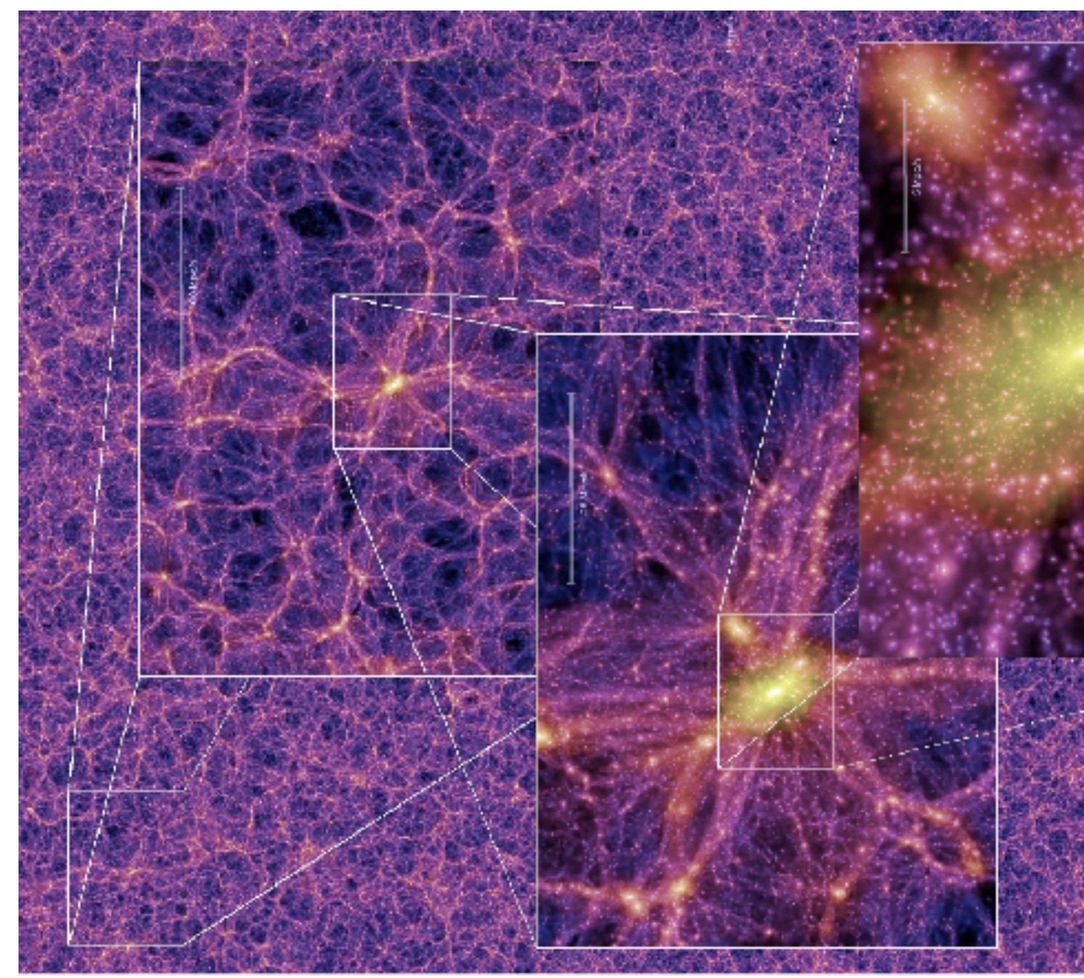


Gil-Marin et al, 2016

How do we compare theory with data?

- If matter density field was Gaussian,
  - PDF of δ is multivariate Gaussian, with diagonal covariance in Fourier space
- Then all the information would be contained in the power spectrum

$$\langle \delta(\boldsymbol{k}) \delta^*(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}')\, P(k)$$



- However, final matter density is clearly non-Gaussian!

Inference beyond the power spectrum

- Assume we observe the matter density field* $\rho(\boldsymbol{x}) = \bar{\rho}[1 + \delta(\boldsymbol{x})]$
- Given cosmological parameters θ, theory predicts

1. Statistics of initial conditions
2. How a given $\delta_{\rm in}(\boldsymbol{x})$ evolves into the final density field

Inference beyond the power spectrum

- Assume we observe the matter density field* $\rho(\boldsymbol{x}) = \bar{\rho}[1 + \delta(\boldsymbol{x})]$
- Given cosmological parameters θ, theory predicts

1. Statistics of initial conditions   Prior $P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$
2. How a given $\delta_{\rm in}(\boldsymbol{x})$ evolves into the final density field

Inference beyond the power spectrum

- Assume we observe the matter density field* $\rho(\boldsymbol{x}) = \bar{\rho}[1 + \delta(\boldsymbol{x})]$
- Given cosmological parameters θ, theory predicts

1. Statistics of initial conditions — Prior $P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$
2. How a given $\delta_{\rm in}(\boldsymbol{x})$ evolves into the final density field

Conditional probability, in absence of errors:

$$P\left(\vec{\delta}\middle|\vec{\delta}_{\rm in}, \theta\right) = \delta_D^{\infty}\left(\vec{\delta} - \vec{\delta}_{\rm fwd}[\vec{\delta}_{\rm in}, \theta]\right)$$

Inference beyond the power spectrum

- For the situation we are dealing with in cosmology, then, the *full posterior of cosmological parameters given the data* is then given by

$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in}\, P\left(\vec{\delta}_{\rm obs}\middle|\vec{\delta}_{\rm in}, \theta\right) P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$$

Inference beyond the power spectrum

- For the situation we are dealing with in cosmology, then, the *full posterior of cosmological parameters given the data* is then given by

$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in}\, P\left(\vec{\delta}_{\rm obs}\middle|\vec{\delta}_{\rm in},\theta\right) P_{\rm prior}\left(\vec{\delta}_{\rm in},\theta\right)$$

Functional integral...

Multivariate Gaussian

Inference beyond the power spectrum

$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in}\, P\left(\vec{\delta}_{\rm obs}\middle|\vec{\delta}_{\rm in}, \theta\right) P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$$

- How does this work in practice? Markov Chain Monte Carlo:
  - Discretize field on grid
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Compare with data and repeat
- Challenge: even with coarse resolution, have to sample many millions of parameters
- Key: Hamiltonian Monte Carlo

Inference beyond the power spectrum

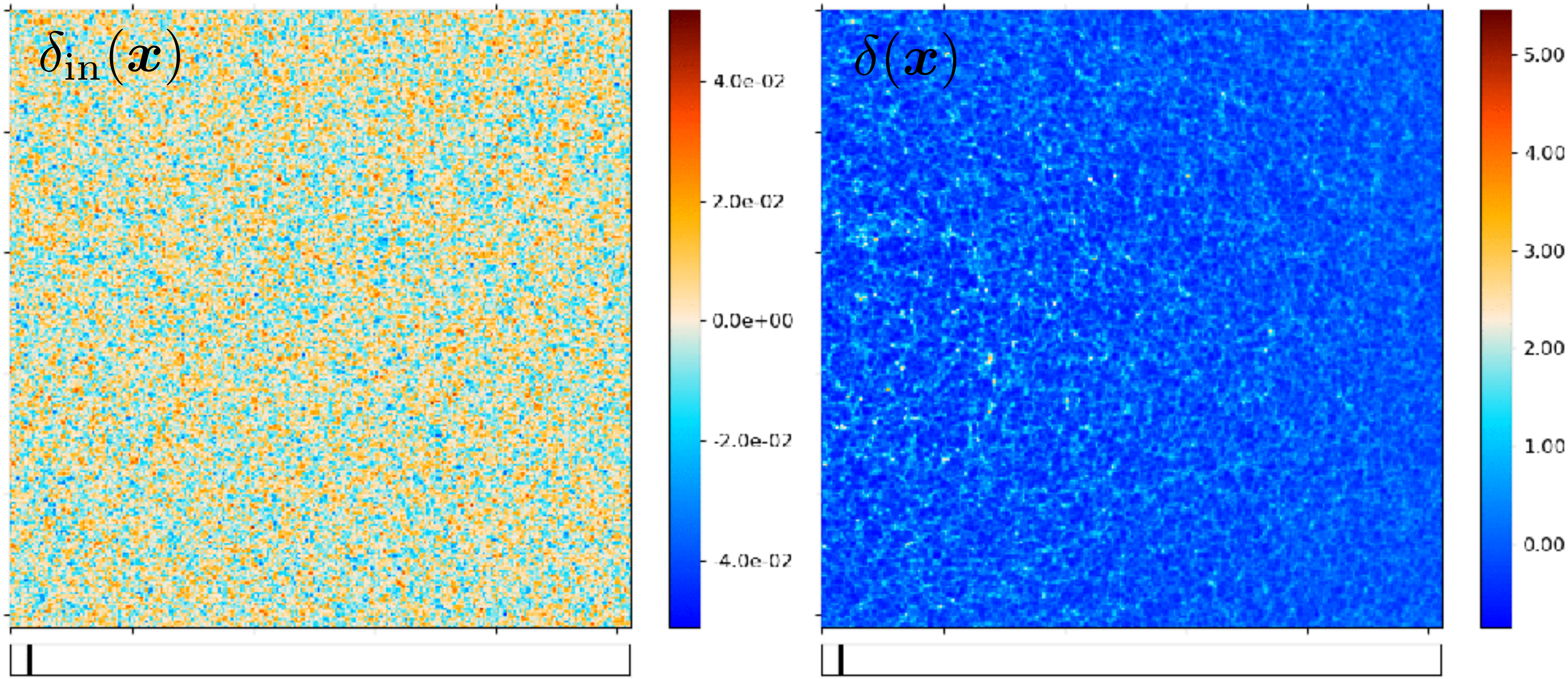
$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in}\, P\left(\vec{\delta}_{\rm obs}\middle|\vec{\delta}_{\rm in}, \theta\right) P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$$

- How does this work in practice? Markov Chain Monte Carlo:
  - Discretize field on grid
  - Draw initial conditions from prior
  - Forward-evolve using gravity
  - Compare with data and repeat

- Lots of interest in this approach recently

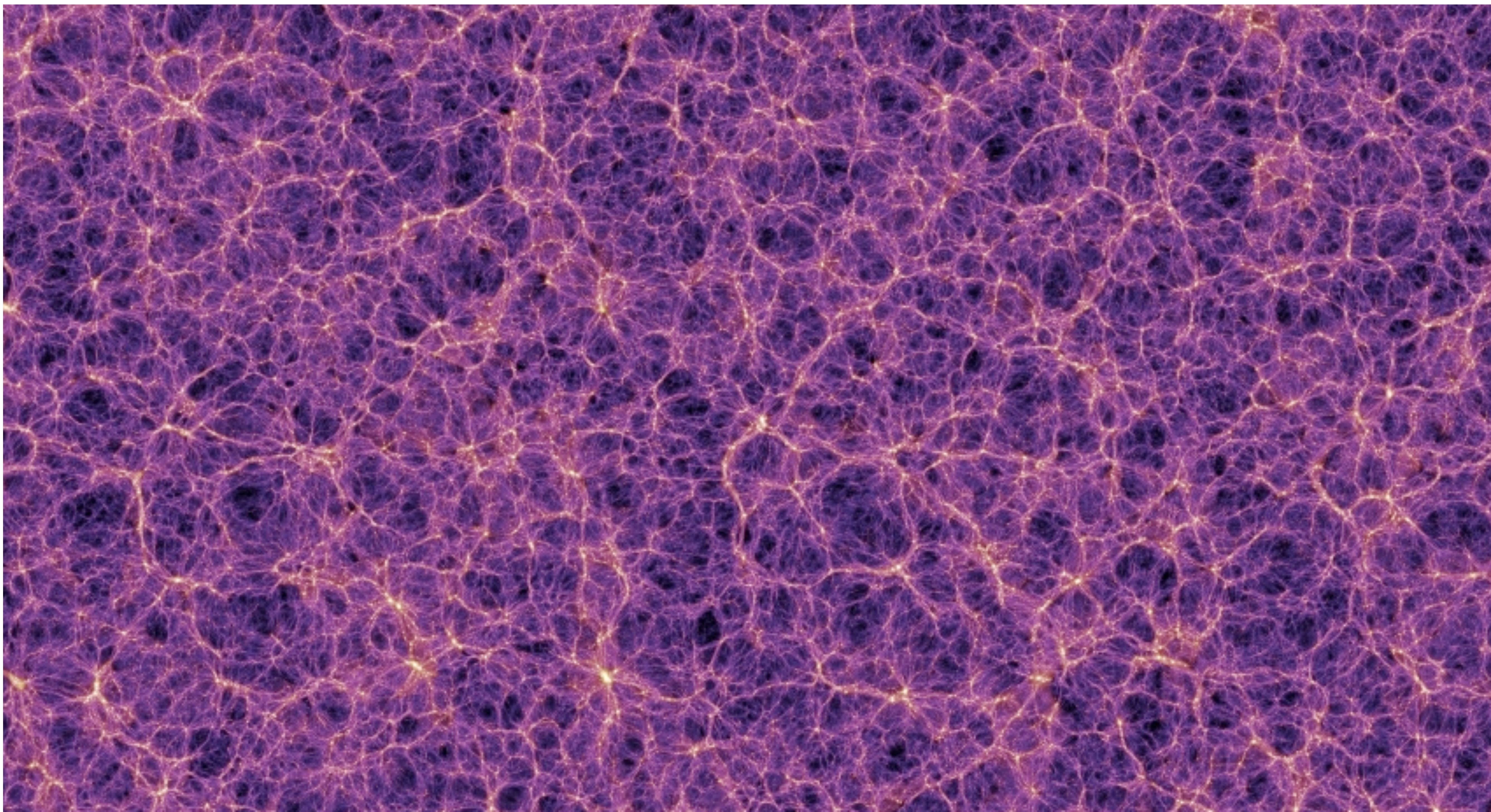
Kitaura & Ensslin, Jasche & Wandelt, Wang, Mo et al, Seljak et al, Jasche & Lavaux (2017), …

Full Bayesian inference in practice

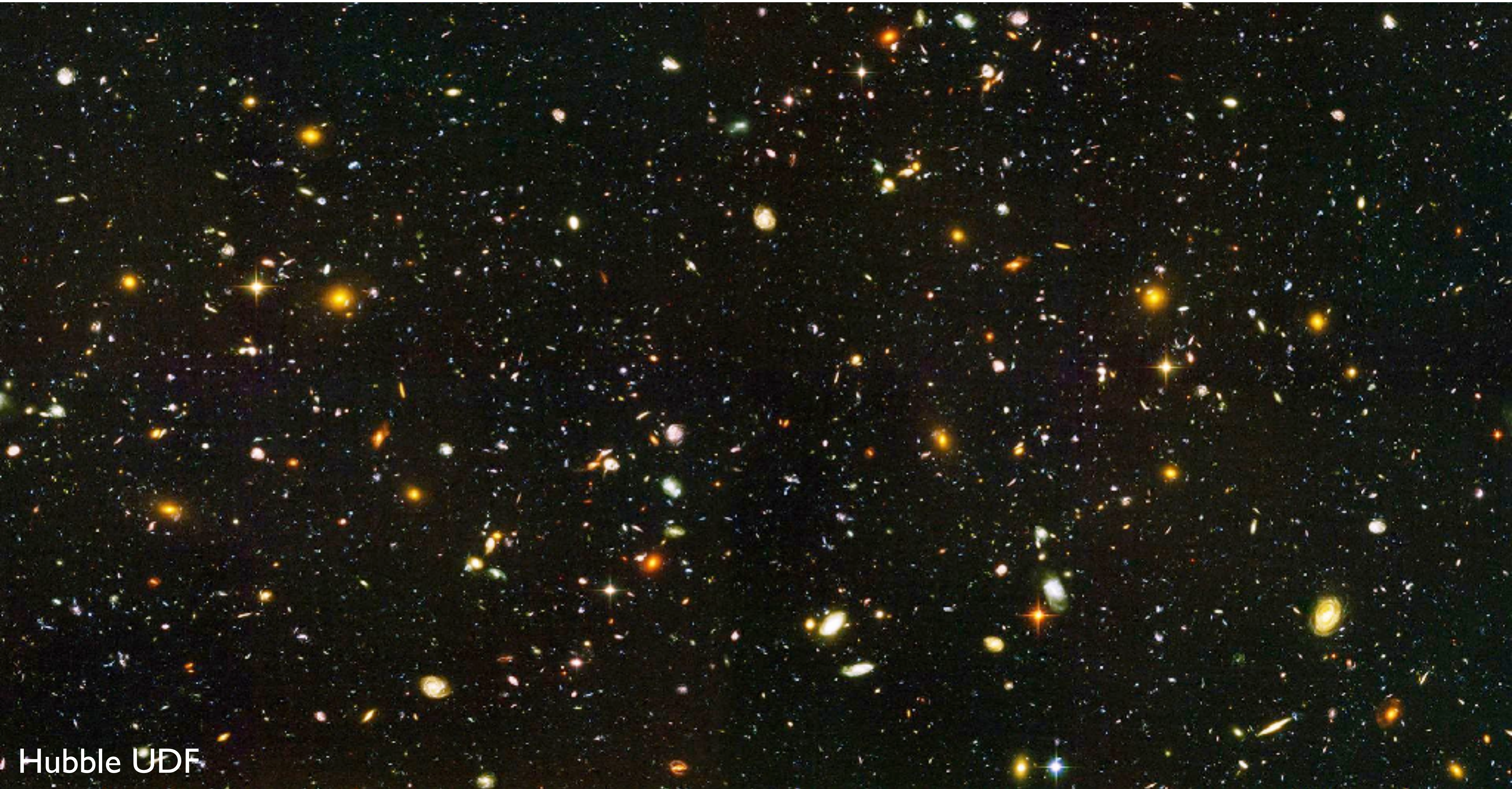


Jasche, Lavaux+ in prep.

We don’t observe the matter distribution, however…

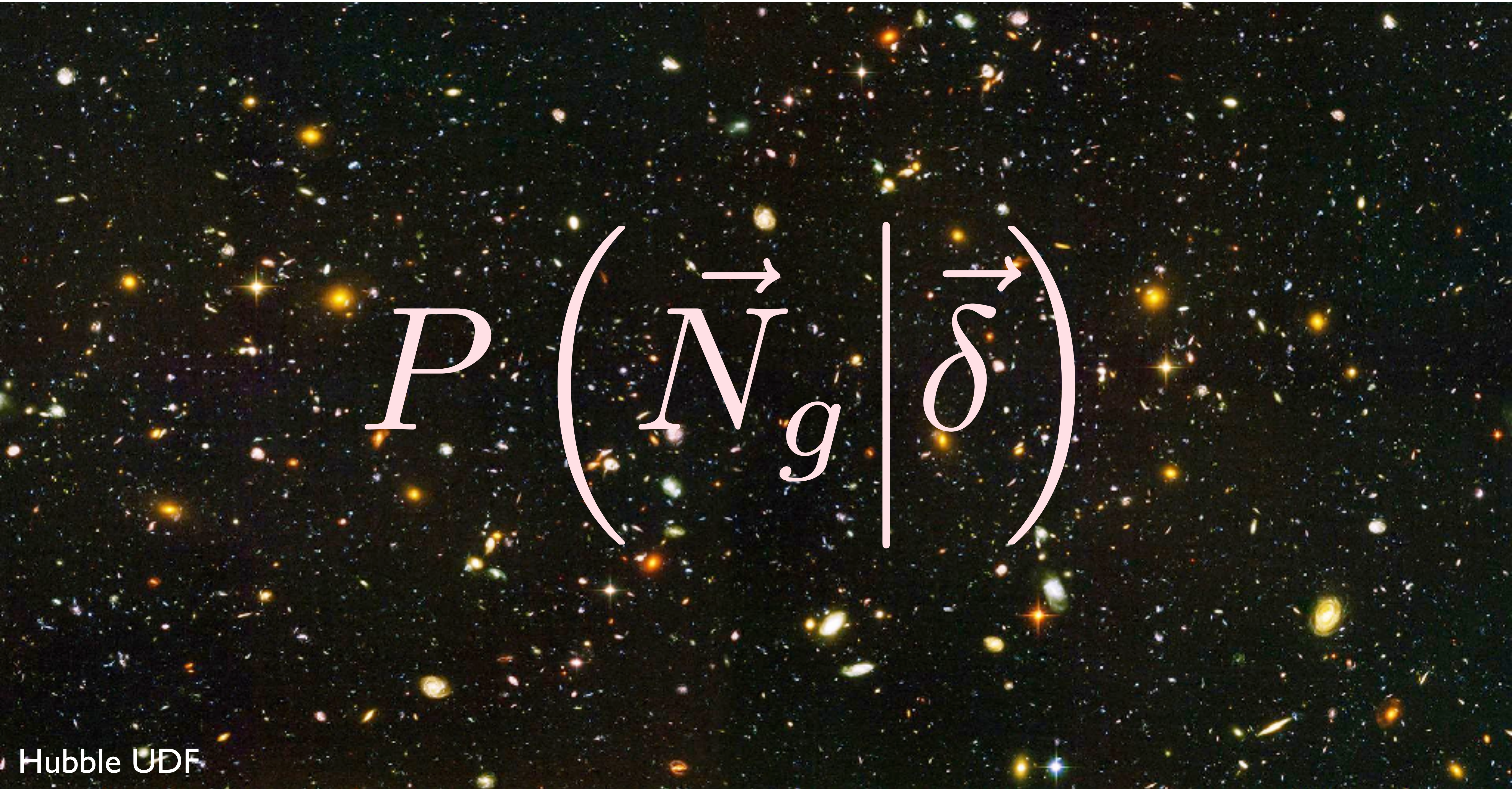


What we observe, is this…



Thus, we need

$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in} \int \mathcal{D}\vec{\delta}\, P\left(\vec{N}_g \middle| \vec{\delta}\right) P\left(\vec{\delta} \middle| \vec{\delta}_{\rm in}, \theta\right) P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$$



Theory of galaxy clustering

- We cannot yet simulate the formation of galaxies* fully realistically
- Need to abstract from the incomplete understanding on small scales
  - Only hope for rigorous results is on scales $k < k_{NL}$
- Goal: describe galaxy clustering up to a given scale and accuracy using a finite number of free bias parameters b_O, and stochastic amplitudes

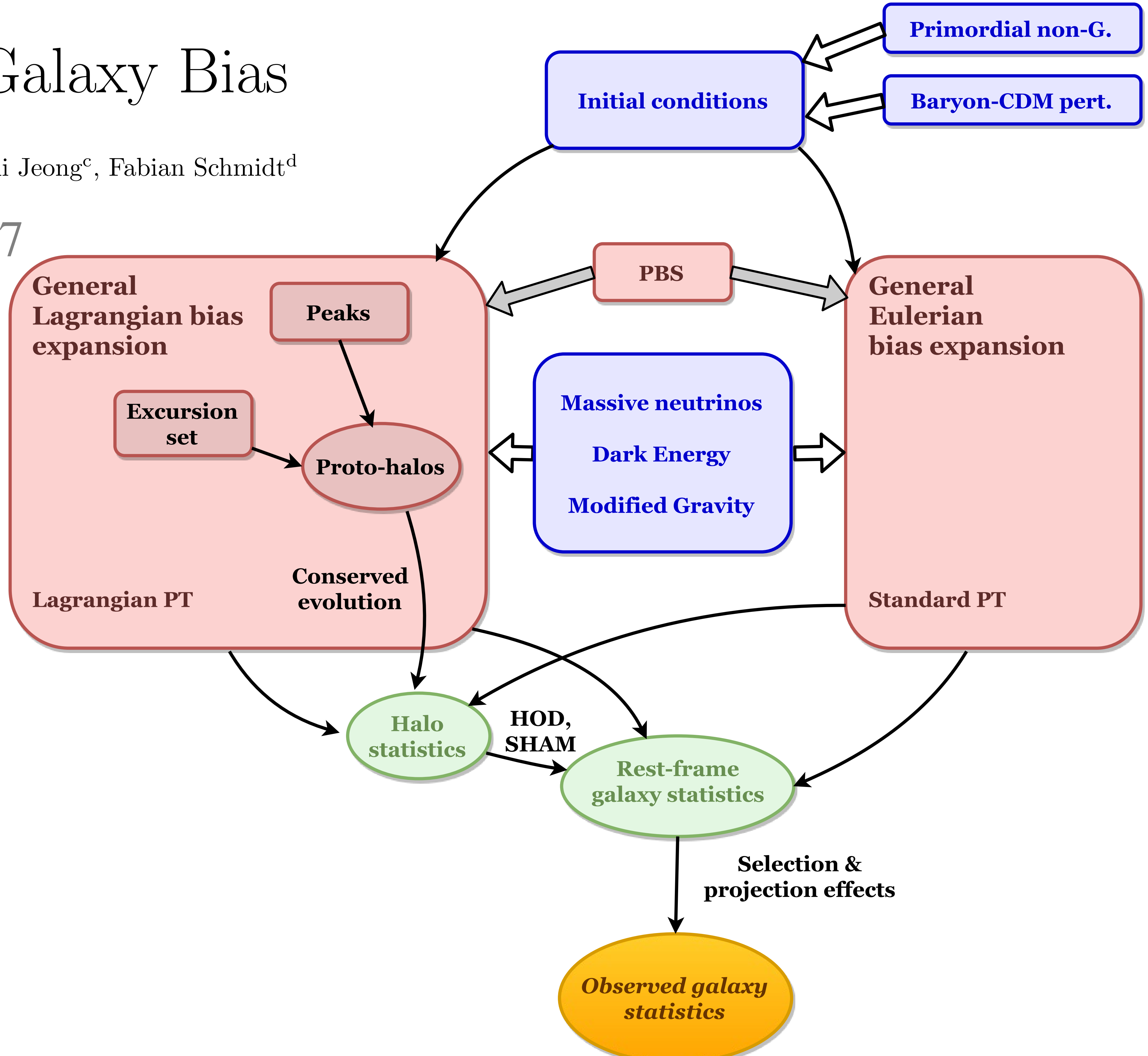
* Everything in following will apply to any tracer of LSS.

Most of what I will talk about, and much more, can be found in:

Large-Scale Galaxy Bias

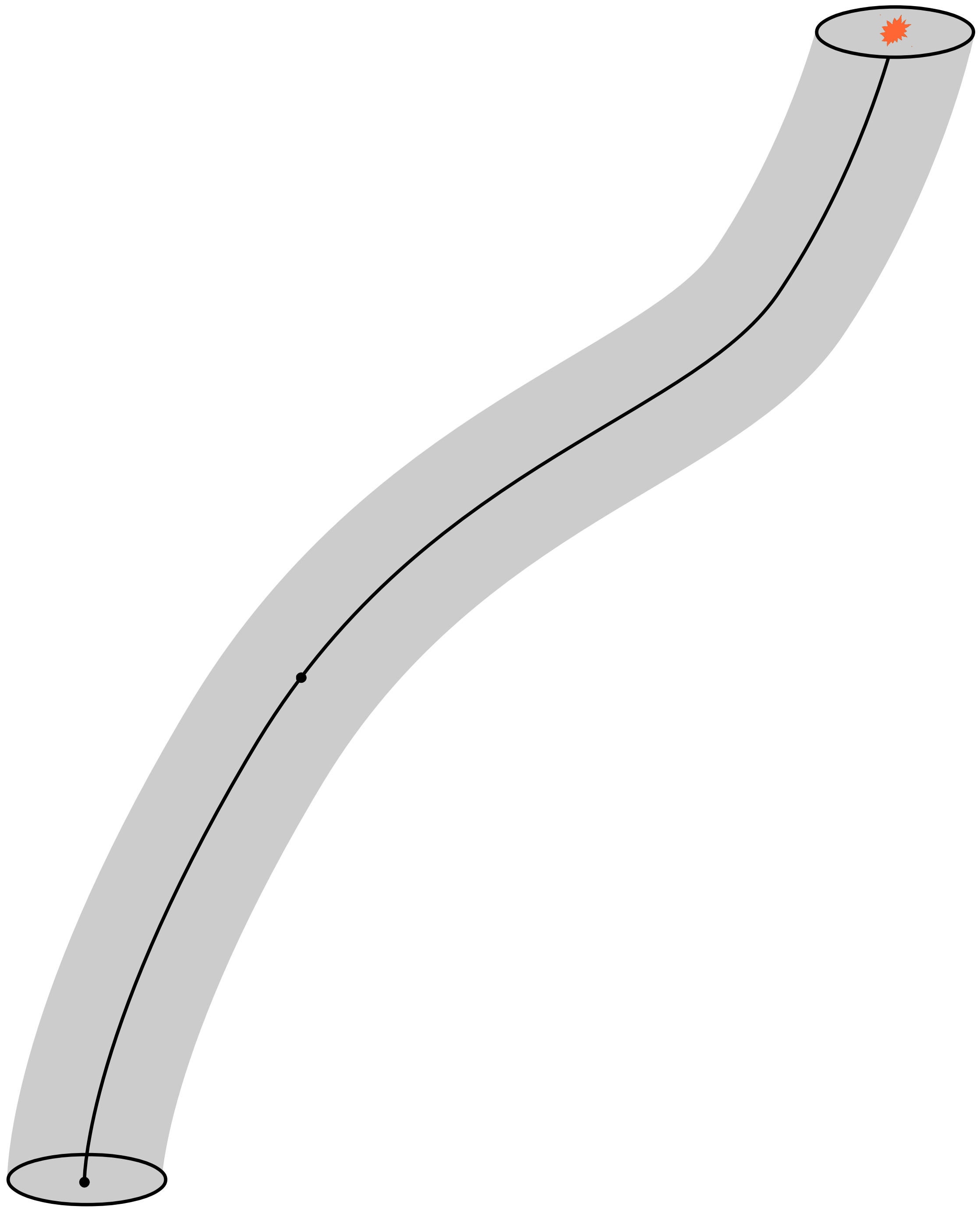
Vincent Desjacques[a,b], Donghui Jeong[c], Fabian Schmidt[d]

arXiv:1611.09787



Galaxy formation

- Consider coarse-grained (large scale) view of region that forms a galaxy at conformal time τ
- Formation happens over *long time scale*, but *small spatial scale* R_*
  - For halos, expect $R_* \lesssim R_L$
- Thus, a gradient expansion makes sense on large scales (small wavenumbers *k*)



EFT approach in LSS

- Effective field theory: write down all terms (in Lagrangian or equations of motion) that are consistent with symmetries
  - Gravity: general covariance
  - Galaxy density: 0-component of 4-vector (momentum density)

EFT approach in LSS

- Effective field theory: write down all terms (in Lagrangian or equations of motion) that are consistent with symmetries
  - Gravity: general covariance
  - Galaxy density: 0-component of 4-vector (momentum density)
- Order contributions by perturbative order, and number of spatial derivatives (gradient expansion)

EFT approach in LSS

- For large-scale structure (LSS), general covariance boils down to the statement that $\Phi, \ \nabla\Phi$ and v cannot appear in bias expansion
  - No surprise: leading gravitational observable is tidal field $\partial_i\partial_j\Phi$

EFT bias expansion

- Can we write the EFT as local in time and space?
  - Only makes sense if spatial and time derivatives are suppressed
- True for spatial derivatives, but not for time derivatives! Galaxies form over many Hubble times (as does matter field)
  - Fundamental theory is *nonlocal in time*

Non-locality in time

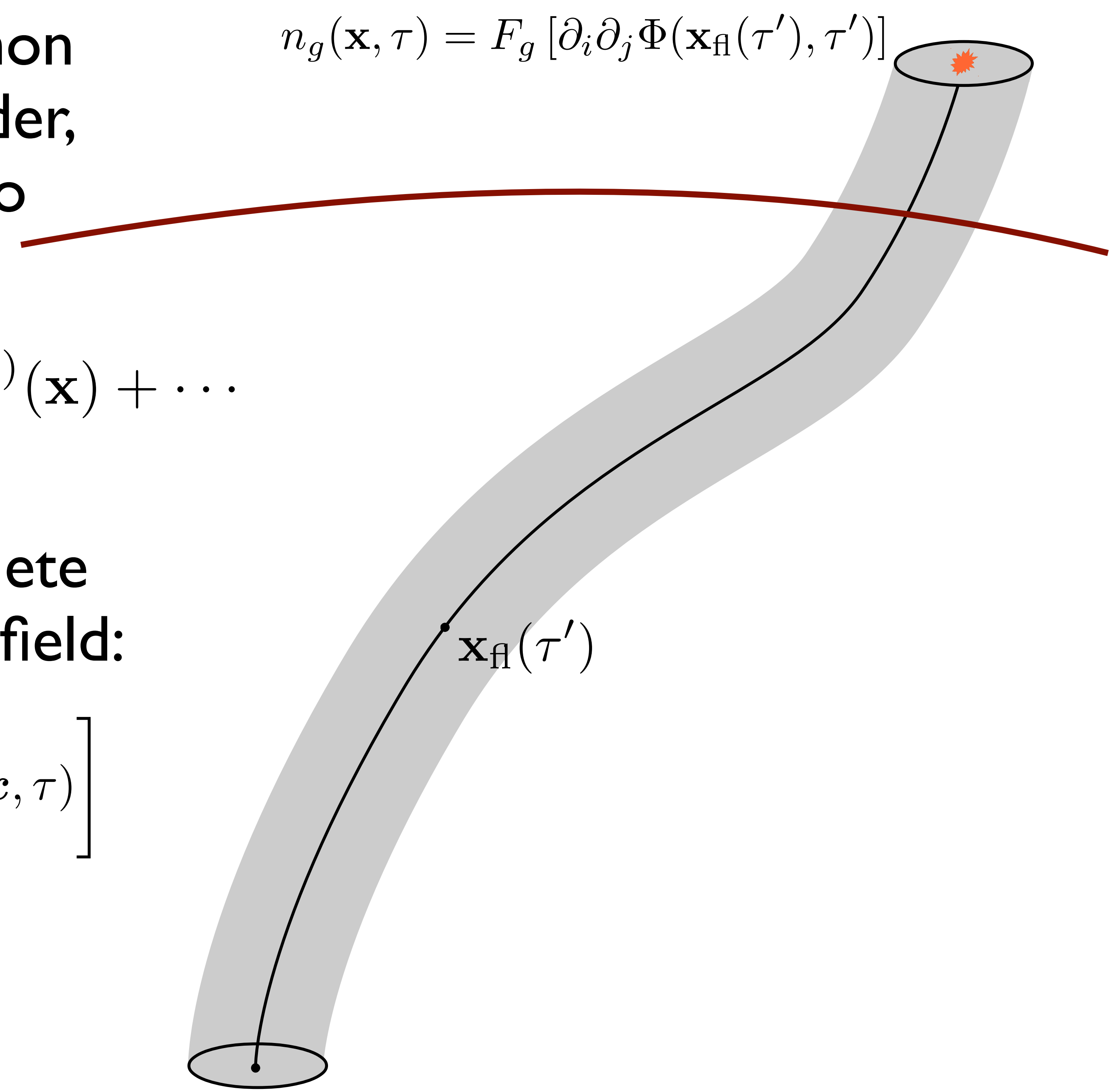
- We can similarly deal with non locality in time at higher order, since expansion continues to factorize:

$$\delta(\mathbf{x},\tau) = D(\tau)\delta^{(1)}(\mathbf{x}) + D^2(\tau)\delta^{(2)}(\mathbf{x}) + \cdots$$

- Allows us to obtain a complete expansion of galaxy density field:

$$n_g(\boldsymbol{x},\tau) = \bar{n}_g(\tau)\left[1 + \sum_O b_O(\tau) O(\boldsymbol{x},\tau)\right]$$

up to given desired order in perturbations



Mirbabayi, FS, Zaldarriaga '14

Complete bias expansion

$$n_g(\boldsymbol{x},\tau) = \bar{n}_g(\tau)\left[1 + \sum_O b_O(\tau) O(\boldsymbol{x},\tau) + \varepsilon(\boldsymbol{x},\tau) + \varepsilon_\delta(\boldsymbol{x},\tau)\delta(\boldsymbol{x},\tau)\cdots\right]$$

- The picture is not complete yet, since this relation can only hold in a "mean-field" sense
- Small-scale perturbations introduce stochasticity ε (and higher-order terms)
- Cannot predict ε as field, but know the *form of statistics:*

$$\langle\varepsilon(\boldsymbol{k})\varepsilon^*(\boldsymbol{k}')\rangle = (2\pi)^3\delta_D(\boldsymbol{k}-\boldsymbol{k}')\left[P_\varepsilon + k^2 P_\varepsilon^{\{2\}} + \cdots\right]$$

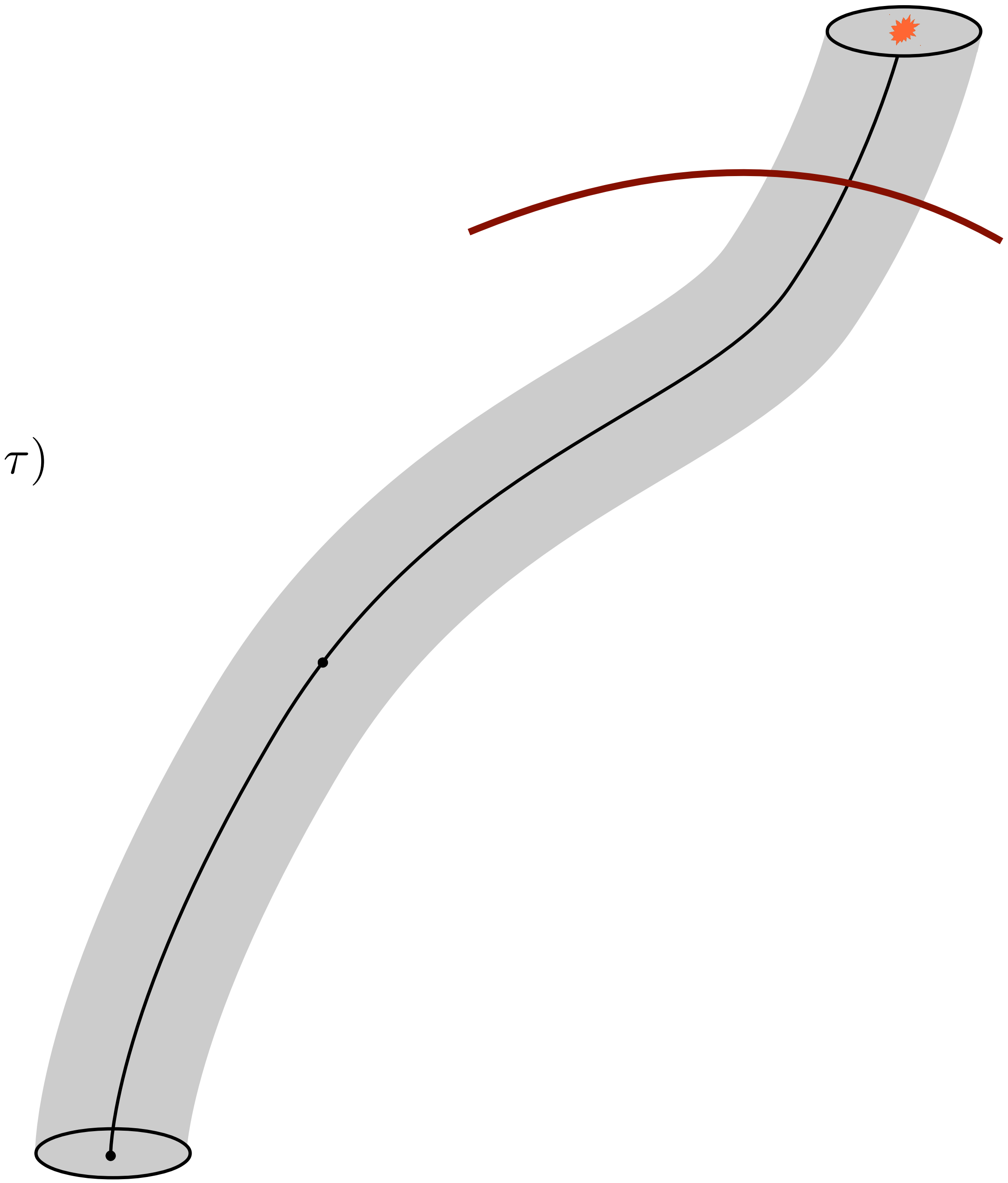
- In the end, stochasticity reduces to fixed number of additional free parameters

Complete bias expansion

- Some virtues of this expansion:
  - Complete in the EFT sense: closed under renormalization
  - Equivalent expansions in Eulerian and Lagrangian space
    - Bias parameters can be mapped from one frame to another unambiguously
  - Allows us to obtain likelihood as well -> later

Spatial nonlocality and scale-dependent bias

- Beyond large-scale limit: need to expand *spatial nonlocality* of galaxy formation
- Higher derivative biases are suppressed with scale R_*
- E.g., $R_*^2 \nabla^2 \delta \longrightarrow \delta_g(\mathbf{k},\tau) = \left(b_1 + b_{\nabla^2\delta} k^2 R_*^2\right) \delta(\mathbf{k},\tau)$
- This also allows for baryonic physics, which *has to come with additional derivatives*
  - Example: pressure perturbations $\delta p = c_s^2 \delta\rho$
  - Pressure force: $\boldsymbol{F} = \boldsymbol{\nabla}\delta p \propto \boldsymbol{\nabla}\delta$
- At higher order in derivatives, time evolution no longer determined by gravity alone



Velocity bias

- Galaxy velocities are important probe of cosmology - but how are they related to matter velocity? $\nabla\Phi$
- Recall that the relative velocity between matter and galaxies is an observable, and thus cannot involve $\Phi,\ \nabla\Phi$, or $\boldsymbol{v}$
- Leading contribution:

$$\boldsymbol{v}_g - \boldsymbol{v}_m = \beta \boldsymbol{\nabla} \delta$$

- Two more derivatives -> suppressed by k^2

Velocity bias

- $\boldsymbol{v}_g - \boldsymbol{v}_m = \beta \boldsymbol{\nabla} \delta$
- This is what we expect from pressure forces

  $\boldsymbol{F} = \boldsymbol{\nabla} \delta p \propto \boldsymbol{\nabla} \delta$

  - Also small-scale stochastic velocities, with power spectrum ~ k^4, which captures virial motions
- Summary: Galaxy velocities are unbiased on large scales
- Can then be used to constrain gravity and dark energy

Application: galaxy power spectrum

- Assume we can measure rest-frame galaxy density
  - That is, neglect redshift-space distortions and other projection effects
- Leading-order galaxy power spectrum at fixed time:

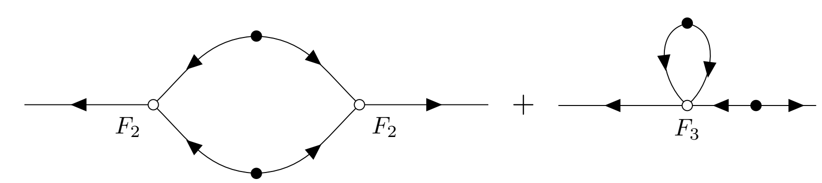
$$P_{gg}(k) = b_1^2 P_{\rm L}(k) + P_\varepsilon^{\{0\}}$$

  - Valid on very large scales
  - 2 free parameters

Application: galaxy power spectrum

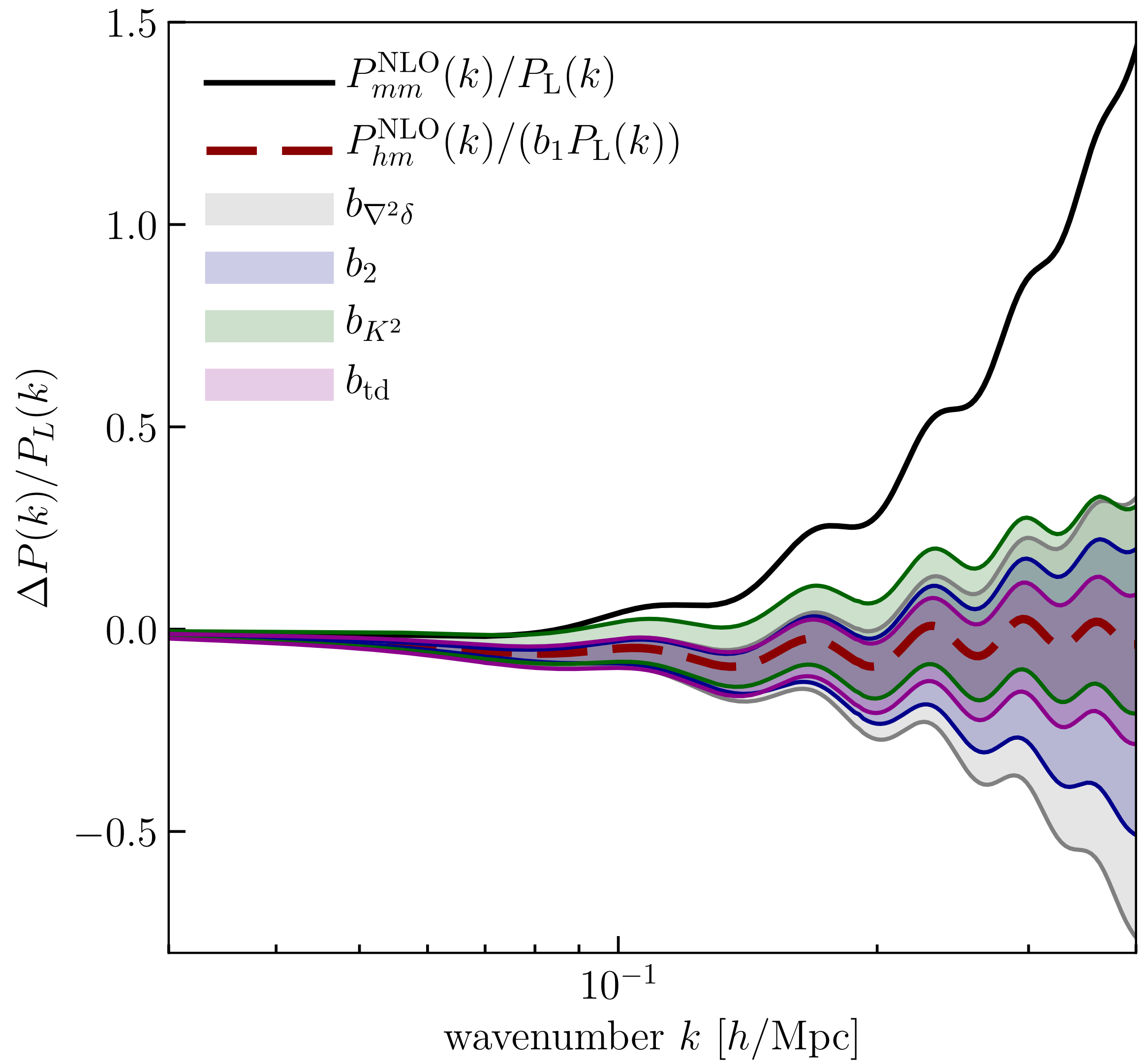
- Next-to-leading order (NLO): involve 2 additional quadratic, 1 cubic, and 2 higher-derivative parameters
  - Quadratic and cubic terms scale like

$$\epsilon_{\rm loop} \equiv \left(\frac{k}{k_{\rm NL}}\right)^{3+n} \approx \left(\frac{k}{0.25\,h\,{\rm Mpc}^{-1}}\right)^{1.3}$$



  - Controlled by shape of P(k) and nonlinear scale
  - Higher-derivative contributions scale as $\epsilon_{\rm deriv.} \equiv k^2 R_*^2$
- Obviously, NLO corrections become important toward smaller scales (higher k)
- Importantly: Two independent expansion parameters!

Fractional size of NLO contributions to galaxy power spectrum



- Many contributions have very similar shape
- If only interested in power spectrum, can significantly reduce number of free parameters
- But then we cannot make use of constraints imposed by the EFT / equivalence principle

Why we *should* go beyond the power spectrum

- Consider galaxy and matter over density at second order:

$$\delta_g^{(2)} = b_1\delta^{(2)} + \frac{1}{2}b_2\delta^2 + b_{K^2}(K_{ij})^2$$
$$\delta^{(2)} = \frac{17}{21}\delta^2 + \frac{2}{7}(K_{ij})^2 - s^k\partial_k\delta$$

  - Equivalence principle ensures that large-scale displacement $\boldsymbol{s} \propto \boldsymbol{\nabla}\Phi$ is the same for galaxies and matter (cf. velocities)
  - Displacement term allows for disentangling bias and amplitude of fluctuations ($\mathcal{A}_s$ or σ_8)
- In terms of summary statistics: need to measure three-point function (bispectrum)
- But there are more such robust terms at higher orders

Full Bayesian inference in the EFT approach

- Recall that we need an expression for the conditional probability $P\left(\vec{N}_g \middle| \vec{\delta}\right)$
- Again, we want to abstract from unknown small-scale details of galaxy formation - likelihood needs to absorb these details
- Describe galaxy counts in term of discretized fractional overdensity: $\vec{N}_g = \bar{N}[1 + \vec{\delta}_g]$

An EFT approach to the likelihood

- Recall that the noise in the galaxy field is approximately Gaussian with analytic power spectrum: $\delta_g(\boldsymbol{k}) - \delta_{g,\mathrm{det}}(\boldsymbol{k}) = \varepsilon(\boldsymbol{k})$

$$\delta_{g,\mathrm{det}}(\boldsymbol{k}) = \sum_O b_O O(\boldsymbol{k})$$

- and $\langle \varepsilon(\boldsymbol{k}) \varepsilon^*(\boldsymbol{k}') \rangle = (2\pi)^3 \delta_D(\boldsymbol{k} - \boldsymbol{k}') \left[P_\varepsilon + k^2 P_\varepsilon^{\{2\}} + \cdots \right]$

- Direct consequence of perturbative expansion

FS, Elsner, et al; 1808:02002

An EFT approach to the likelihood

- Hence, write the conditional probability in Fourier space:

$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in} \int d\{b_O\}\, P\left(\vec{\delta}_g \middle| \vec{\delta}_{\rm fwd}[\vec{\delta}_{\rm in}, \theta], \{b_O\}\right) P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$$

with

$$P\left(\vec{\delta}_g \middle| \delta\right) \propto \left(\prod_{\boldsymbol{k}\neq 0}^{k_{\rm max}} \sigma^2(k)\right)^{-1/2} \exp\left[-\frac{1}{2}\sum_{\boldsymbol{k}\neq 0}^{k_{\rm max}} \frac{1}{\sigma^2(k)} \left|\delta_g(\boldsymbol{k}) - \delta_{g,\rm det}(\boldsymbol{k})\right|^2\right]$$

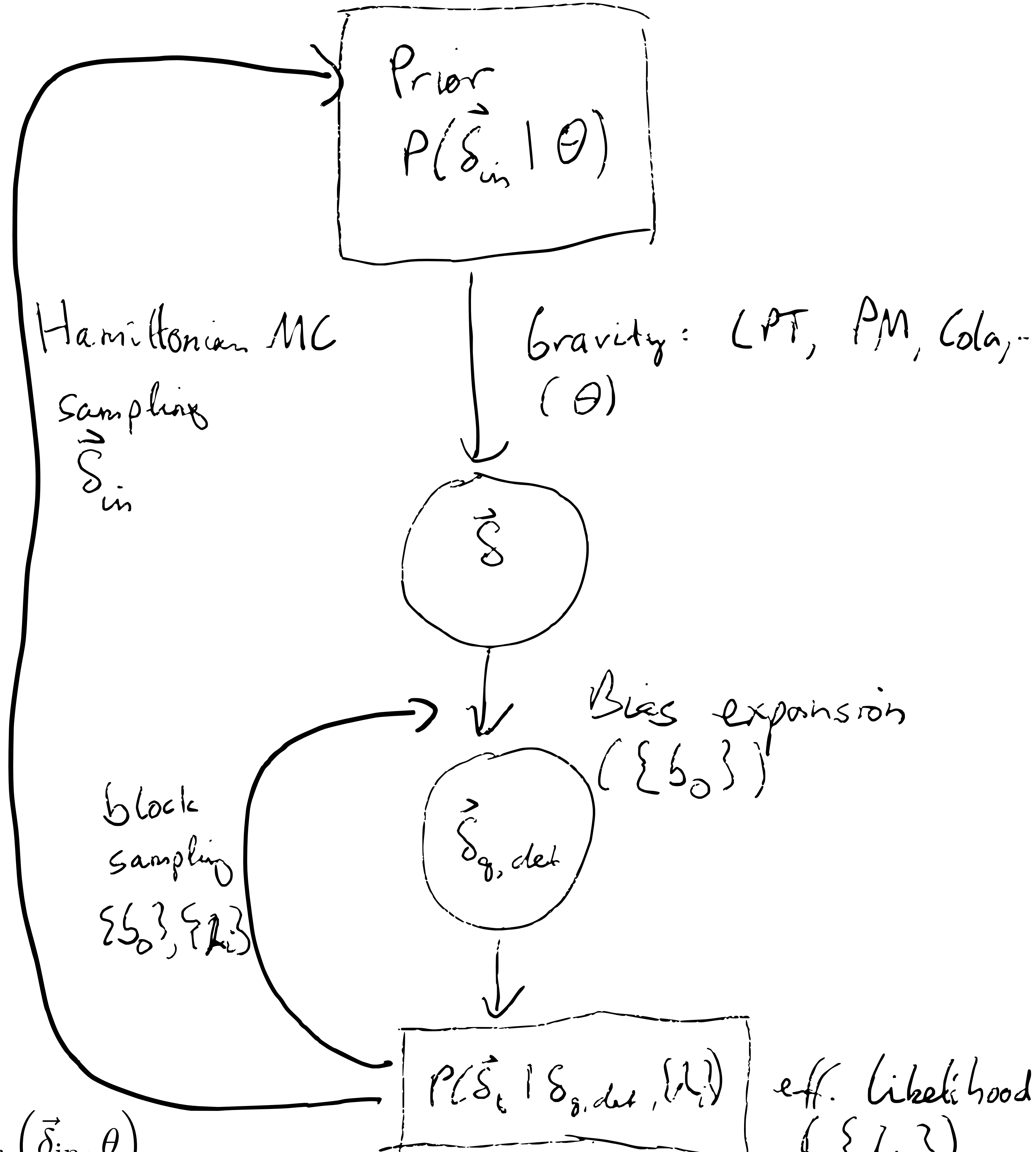
and

$$\delta_{g,\rm det}(\boldsymbol{k}) = \sum_O b_O O(\boldsymbol{k})$$

$$\sigma^2(k) = \sigma_0^2 + \sigma_2^2 k^2 + \sigma_4^2 k^4$$

FS, Elsner, et al; 1808:02002

Flowchart:



$$P(\theta) = \int \mathcal{D}\vec{\delta}_{\rm in} \int d\{b_O\}$$
$$P\left(\vec{\delta}_g \middle| \vec{\delta}_{\rm fwd}[\vec{\delta}_{\rm in}, \theta], \{b_O\}\right) P_{\rm prior}\left(\vec{\delta}_{\rm in}, \theta\right)$$

EFT likelihood implementation

- Concrete implementation: 2LPT forward evolution with

$$O \in \left\{\delta,\ \delta^2 - \left\langle\delta^2\right\rangle,\ (K_{ij}^2) - \left\langle(K_{ij})^2\right\rangle,\ \nabla^2\delta\right\}$$

$$\text{with coefficients} \quad \left\{b_1,\ \frac{b_2}{2},\ b_{K^2},\ c_{\nabla^2\delta}\right\}.$$

(actually, renormalized versions of these operators, constructed from density sharp-k filtered at $2k_{max}$)

- 7 free parameters, plus k_{max} as fixed metaparameter

FS, Elsner, et al; 1808:02002

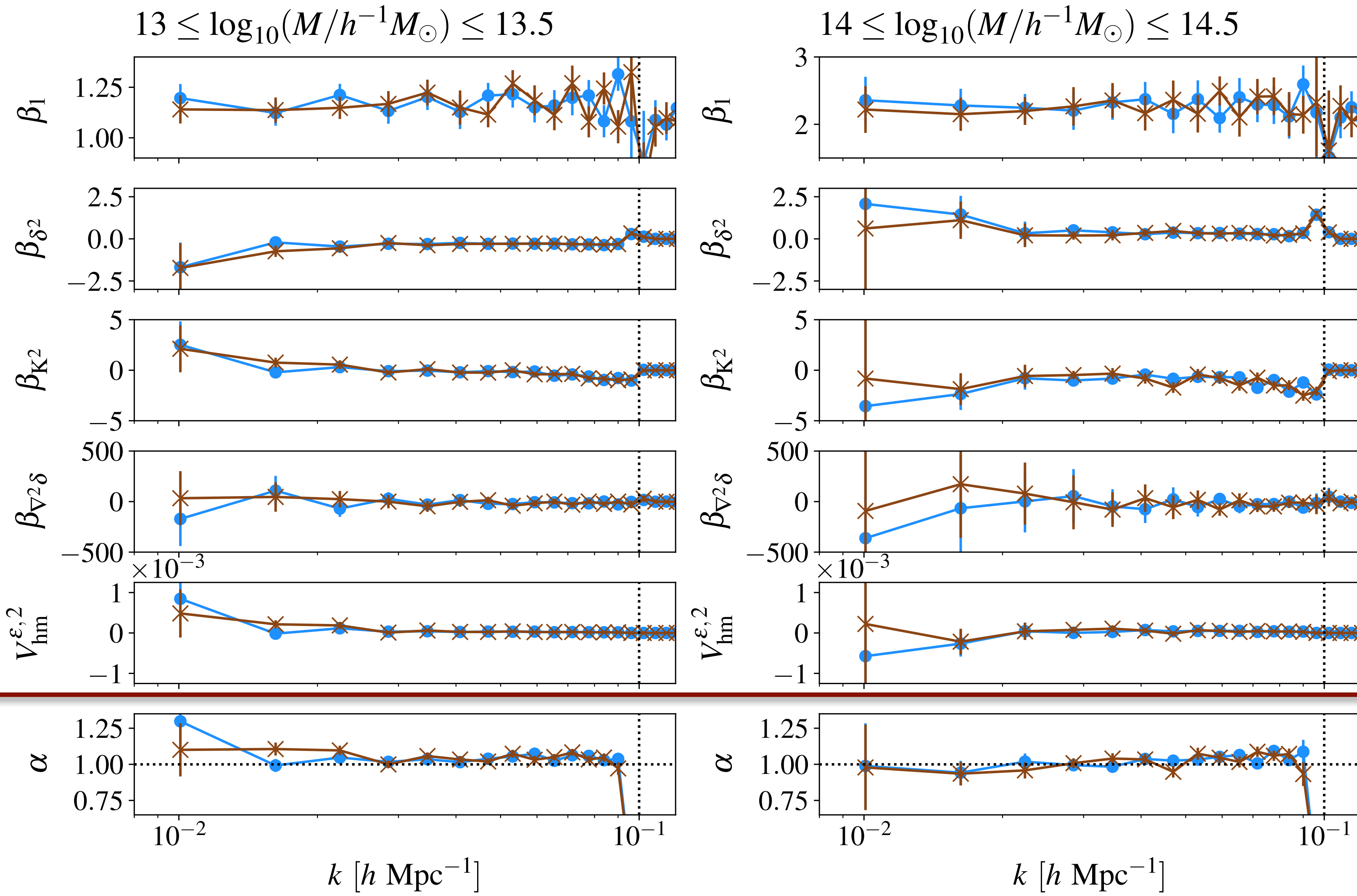
EFT likelihood implementation

- Includes complete bias expansion up to second order and leading higher-derivative term
- Can show that this Fourier-space EFT likelihood captures information equivalent to
  - Galaxy power spectrum at 1-loop order
  - Galaxy bispectrum at leading order
  - Full 2LPT BAO reconstruction

Combined!

FS, Elsner, et al; 1808:02002

First results



$$\alpha \equiv \sigma_8/\sigma_8^{\rm true}$$

$$\alpha[k \leq 0.05\, h\, \mathrm{Mpc}^{-1}] \in [0.99, 1.04]$$

- From halos in N-body simulations
- Right now, phases are fixed to true values

Summary

- LSS contains a wealth of information on dark energy, growth of structure, and the early Universe
- To use this, we need to understand nonlinear (and nonlocal) relation between initial conditions and observed galaxies
- We now have a complete framework for galaxy biasing (on perturbative scales)
  - Also for galaxy velocities
- Leads to well-defined prediction for all *n*-point functions of galaxies
  - "Just compute"
  - Lots of free parameters, however

Summary

- Next challenges:
  - How much information in nonlinear galaxy clustering, given these many free parameters?
  - How best to extract it?
- Right approach in principle: *full Bayesian inference* ("forward modeling") with explicit marginalization over phases
  - We have made recent progress in understanding how the EFT approach provide us with a *robust likelihood* for this purpose