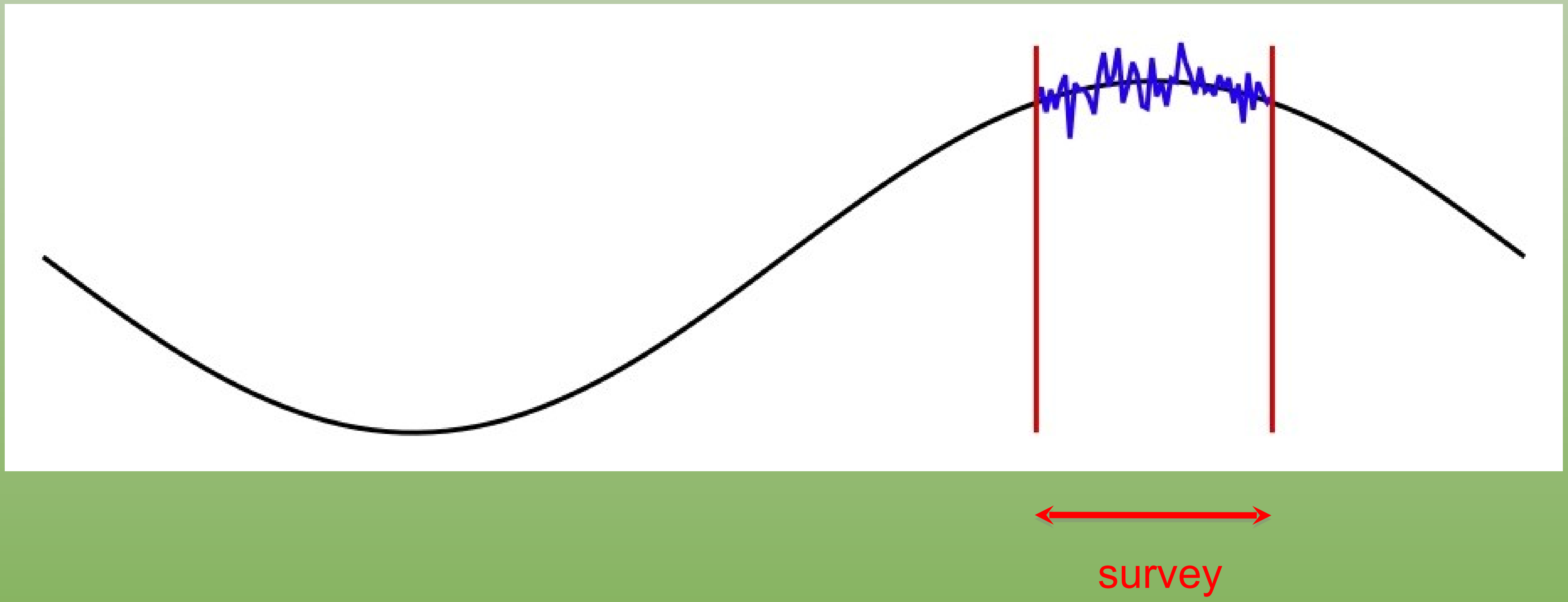
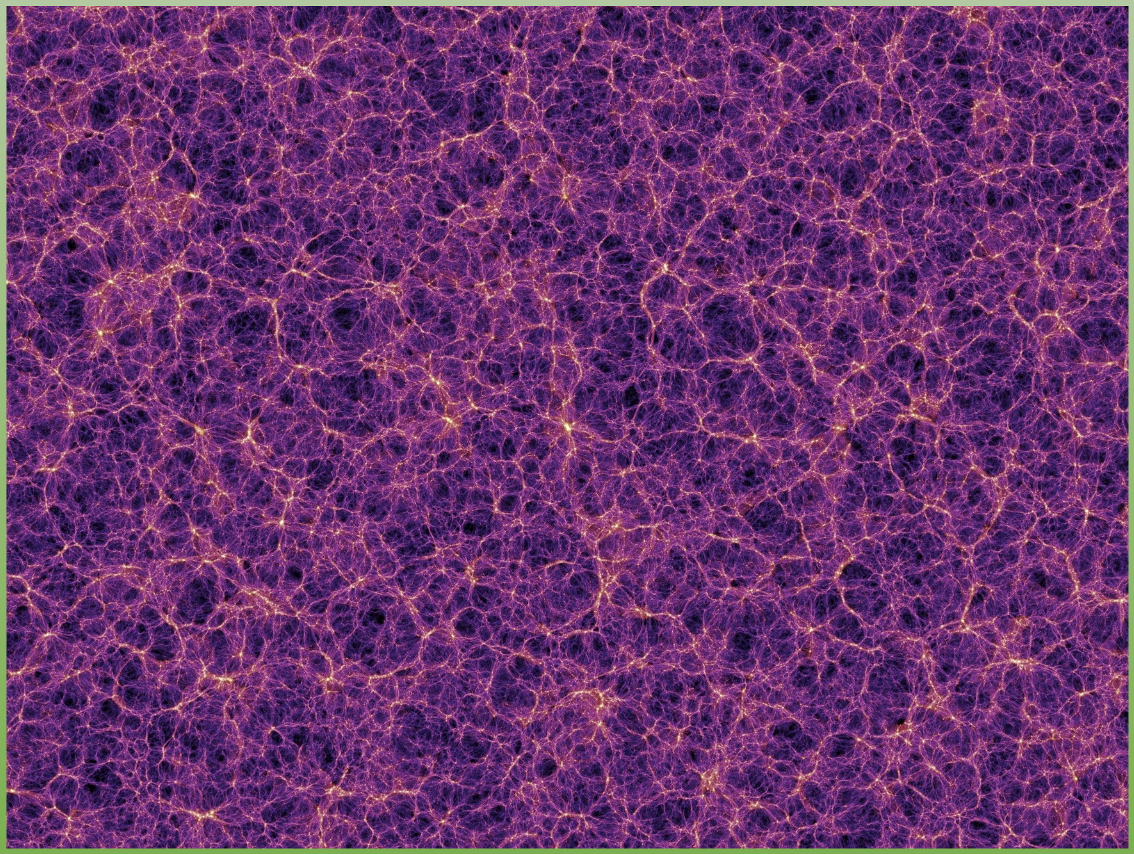
Super-sample covariance



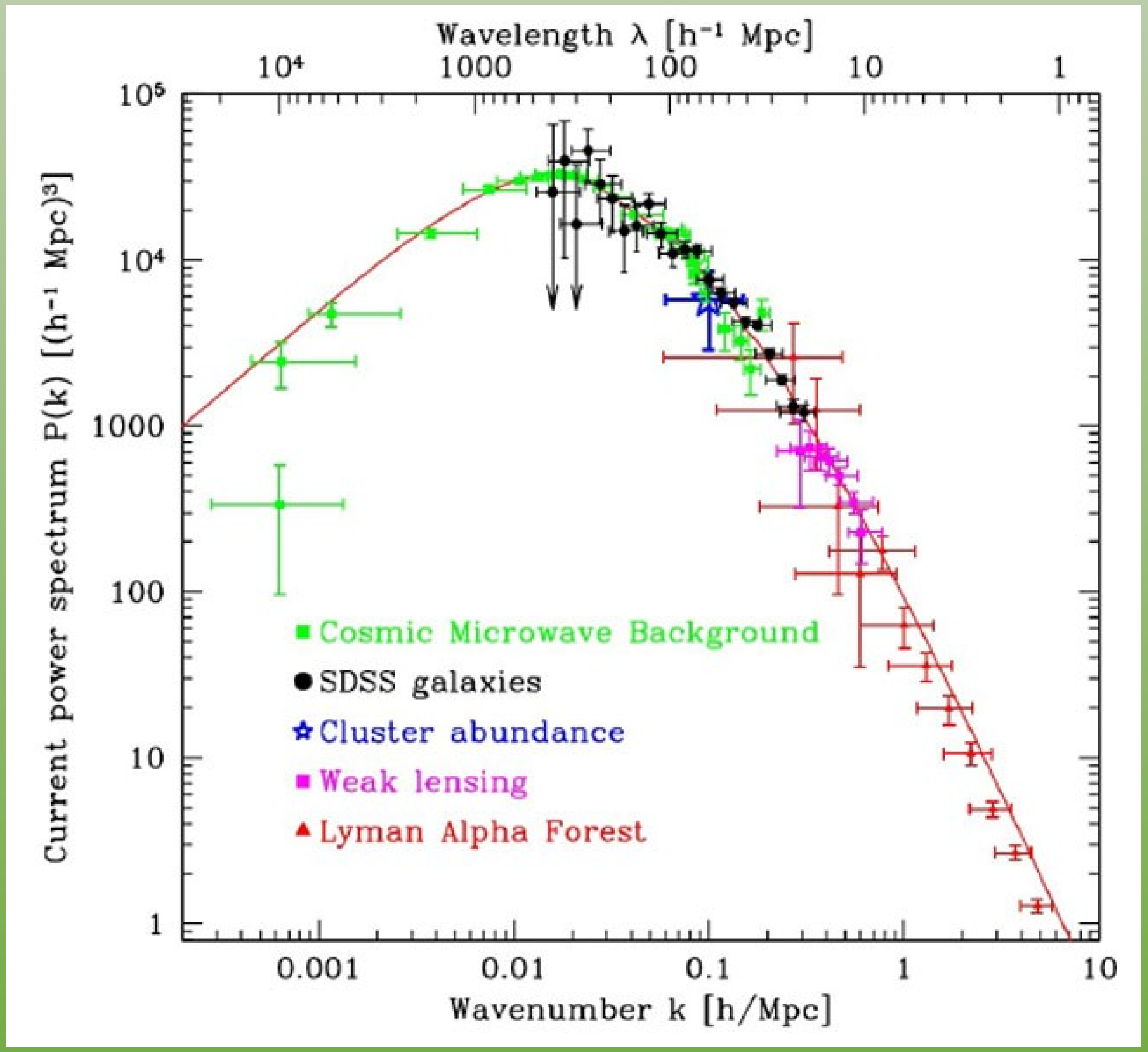
fabien.lacasa@unige.ch

Lacasa & Rosenfeld 2016,
arXiv: 1612.05958, 1703.03337

Density fluctuations on all scales

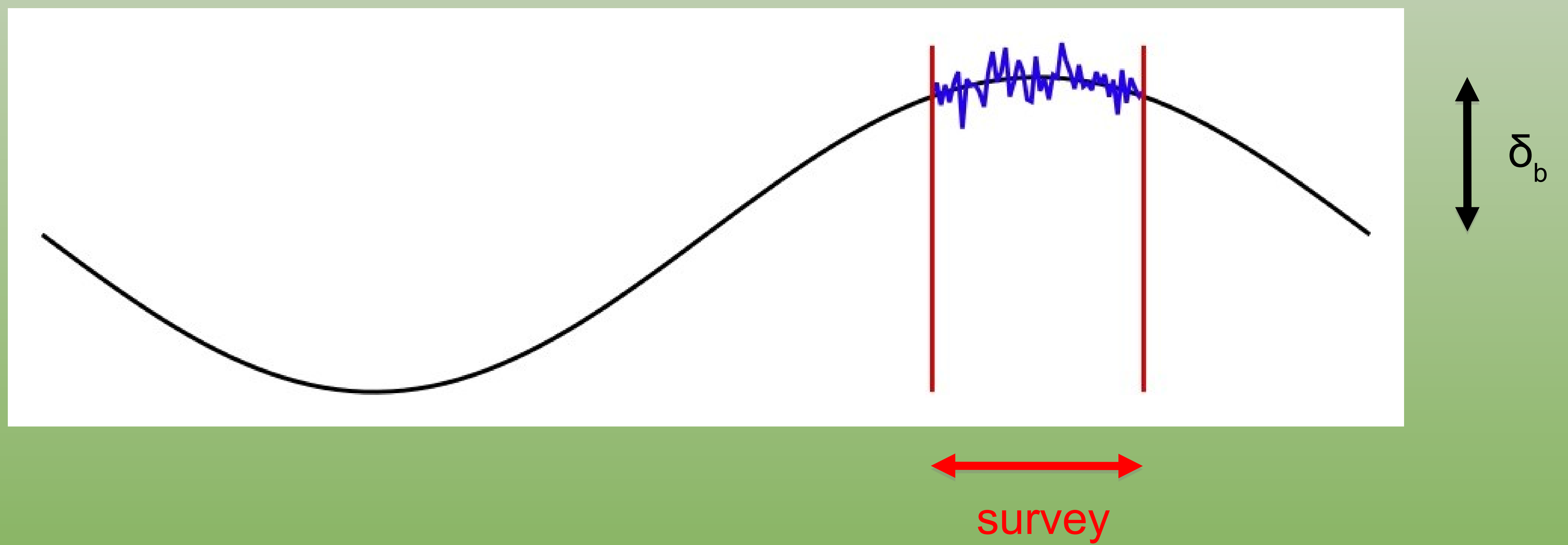


Millenium simulation
(MPA Garching)



Tegmark 2004

Long wave effect



if δ_b>0 : easier to pass δ_{crit} => more halos, especially at high mass

more non-linearity => higher P(k) too for galaxies and shear

SSC derivation

$$\delta_h = b_1\,\delta_b\ (+b_2\delta_b^2 + \cdots)$$

Halos are biased w.r.t. matter

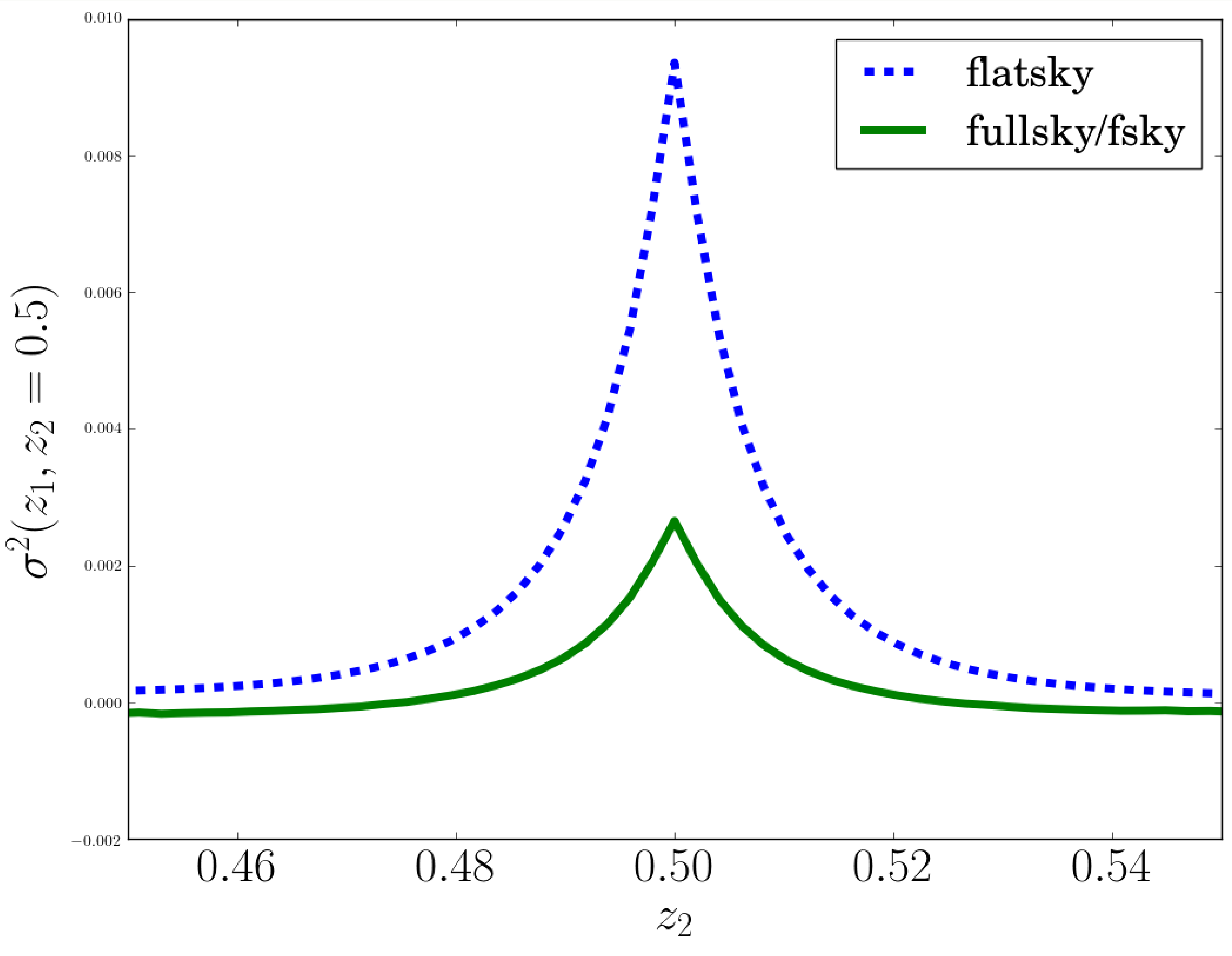
$$\delta_h = \frac{n_h - \overline{n}_h}{\overline{n}_h}$$

Number of halos : $N_h(i_M, i_z) = \int \mathrm{d}M\,\mathrm{dV}\ n_h$

$$\mathrm{Cov}\left(N_h(i_M, i_z), N_h(j_M, j_z)\right) = \int \mathrm{d}M_1 \mathrm{d}M_2\,\mathrm{d}V_1 \mathrm{d}V_2\ \overline{n}_h(M_1, z_1) b_1(M_1, z_1) \times \overline{n}_h(M_2, z_2) b_1(M_2, z_2)\ \sigma^2_{\mathrm{survey}}(z_1, z_2)$$

Covariance of the background density $\sigma^2(z_1, z_2) = \mathrm{Cov}\left(\delta_b(z_1), \delta_b(z_2)\right)$

Covariance of background density



Lacasa, Lima & Aguena
arXiv:1612.05958

SSC approximations I

$$\mathrm{Cov}\left(N_h(i_M,i_z),N_h(j_M,j_z)\right)=\int \mathrm{d}M_1\mathrm{d}M_2\,\mathrm{d}V_1\mathrm{d}V_2\;\overline{n}_h(M_1,z_1)b_1(M_1,z_1)$$
$$\times\;\overline{n}_h(M_2,z_2)b_1(M_2,z_2)\;\sigma^2_{\mathrm{survey}}(z_1,z_2)$$

- Approximation 1 : mass function and bias vary slowly with redshift (compared to σ^2)

$$\mathrm{Cov}_{\mathrm{SSC}}\approx N_h(i_M,i_z)\,b_1(i_M,i_z)\;N_h(j_M,j_z)\,b_1(j_M,j_z)\times S_{i_z,j_z}$$

e.g. Aguena & Lima 2016

- Approximation 2 : radial bin width >> perpendicular survey extension

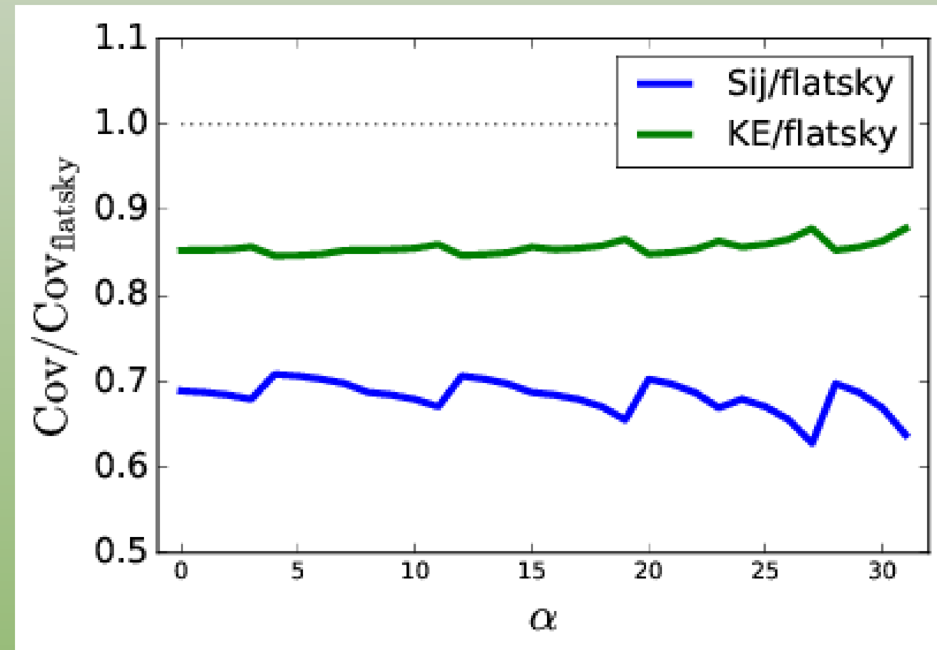
$$\mathrm{Cov}_{\mathrm{SSC}}\approx\delta_{i_z,j_z}\int \mathrm{d}V r(z)^2\;n_h(i_M,z)b_1(i_M,z)\;n_h(j_M,z)b_1(j_M,z)\;\sigma_b(\Omega_S,z)$$

e.g. Krause & Eifler 2016

SSC approximations II

2 redshift bins
z=0.4-0.6
Dz=0.1

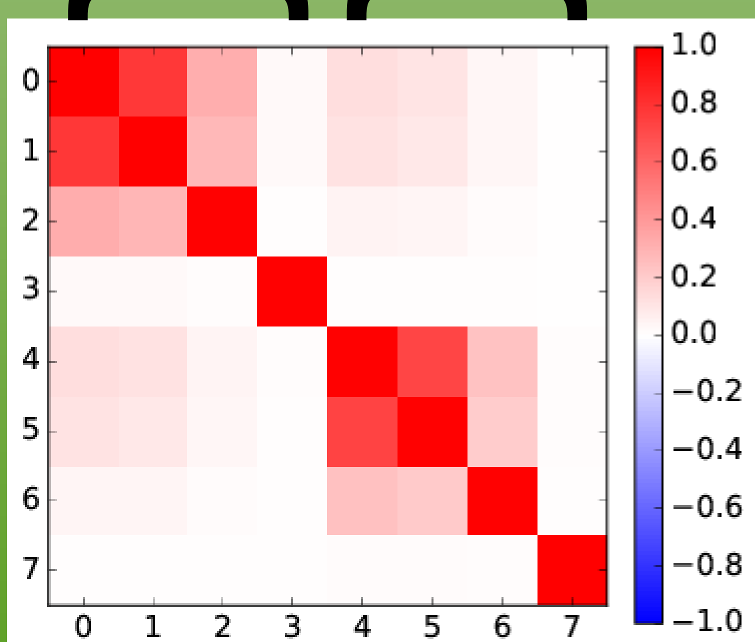
4 mass bins
LogM = 14-16
DlogM = 0.5



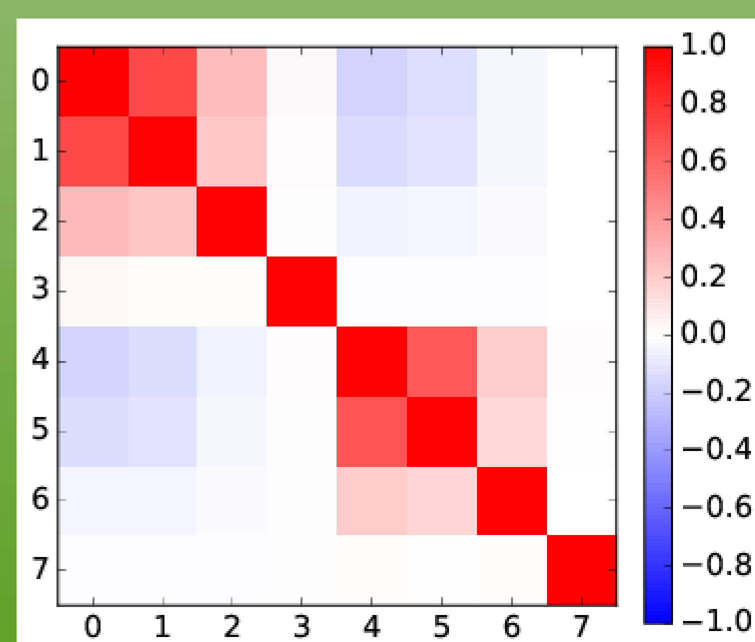
Ratio approx/full
for auto-z

Lacasa, Lima & Aguena
arXiv:1612.05958

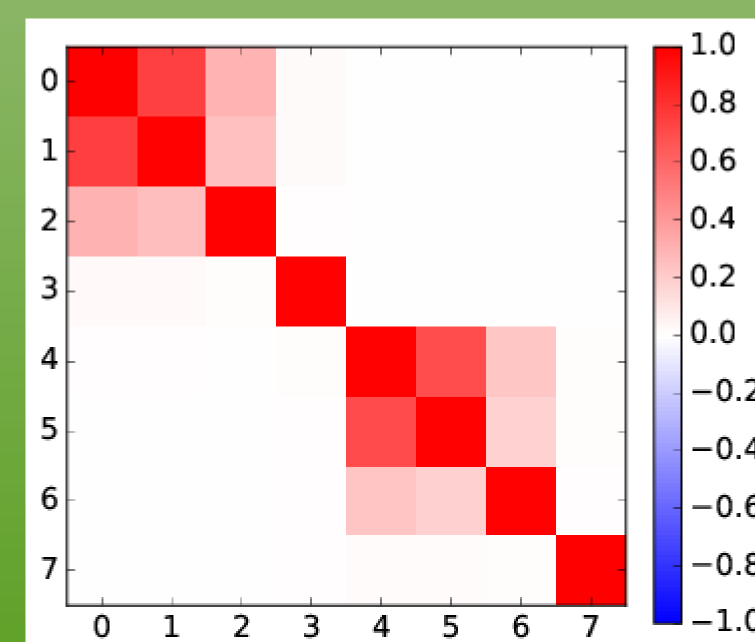
z-bin 1 z-bin 2



full computation



approx 1



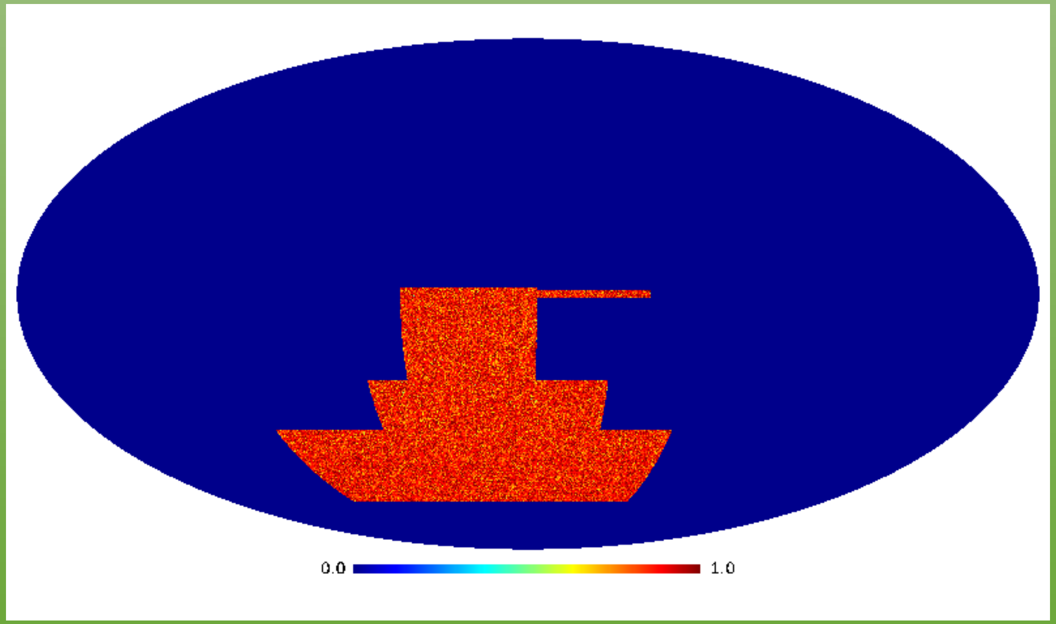
approx 2

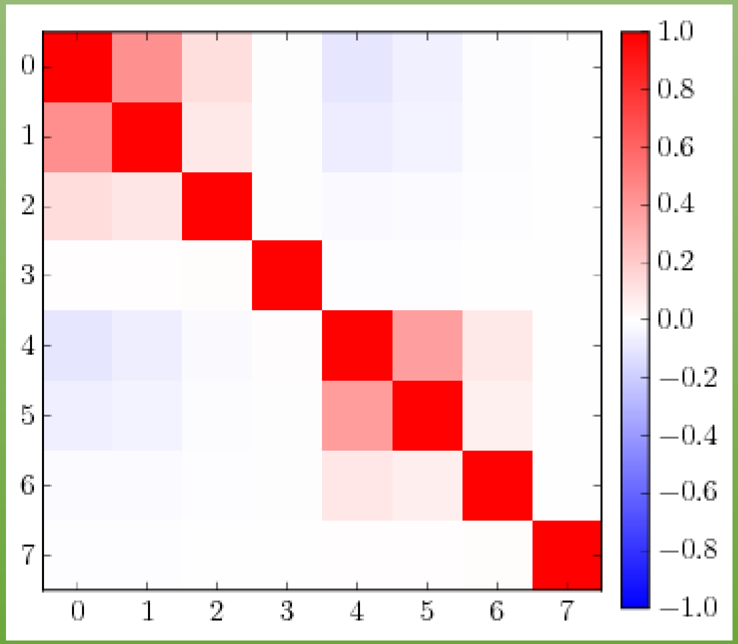
Partial sky / general mask

$$\mathrm{Cov}^{\mathrm{SSC}} = \sum_\ell \frac{2\ell+1}{4\pi} \boxed{C_\ell(W)} \boxed{\mathrm{Cov}_\ell^{\mathrm{SSC}}}$$

Geometry :
mask angular power spectrum

Physics :
observable reaction convolved
with the matter power spectrum





- Holes in the mask (e.g. stars) do not affect SSC
- Up to ~10% anti-correlation between redshift bins (for Dz=0.1)

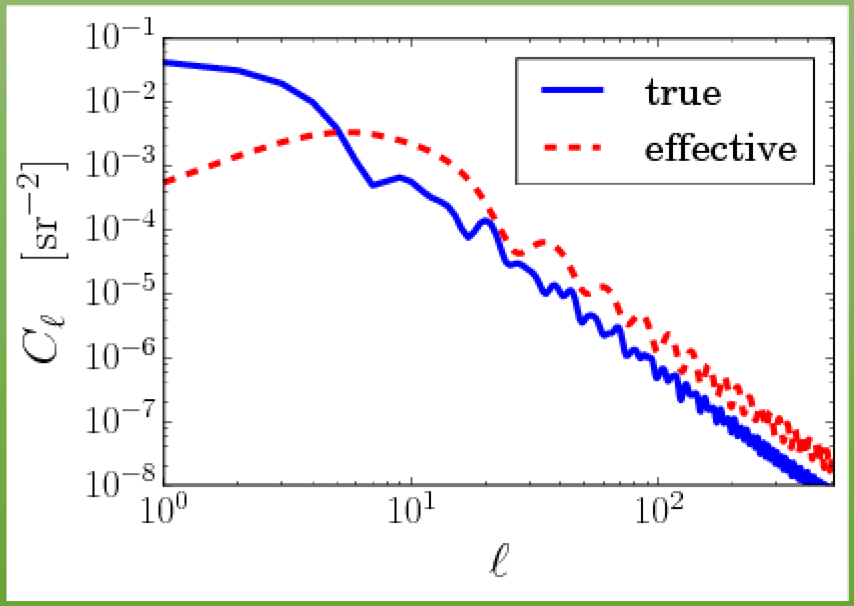
arXiv:1612.05958

Internal covariance estimation

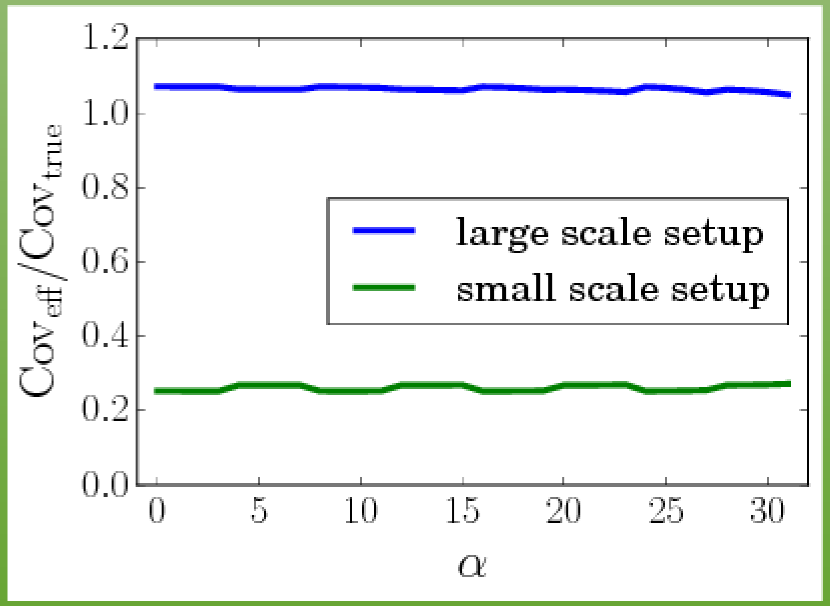
$$< \widehat{\mathrm{Cov}} >= \sum_\ell \frac{2\ell+1}{4\pi} \, C_\ell^{\mathrm{eff}} \, \mathrm{Cov}_\ell^{\mathrm{SSC}}$$

Jackknife/bootstrap is a rescaling of the estimate for the covariance of the subsample
=> unbiased only if subsamples are independent
which is not the case with SSC

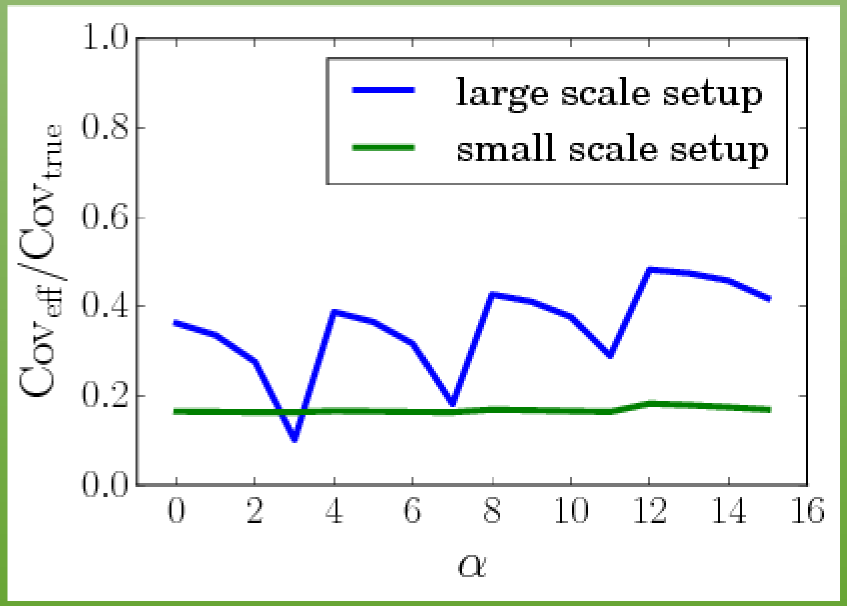
Lacasa & Kunz
arXiv:1703.03337



Mask power spectrum
vs effective from jacknife

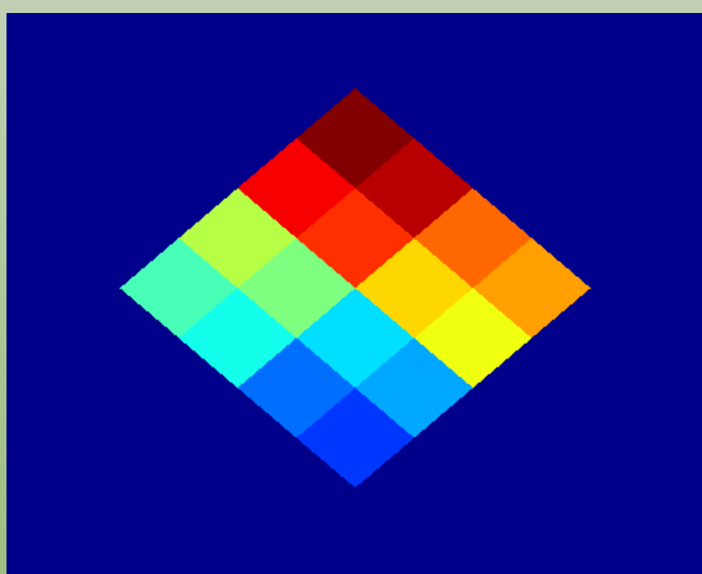


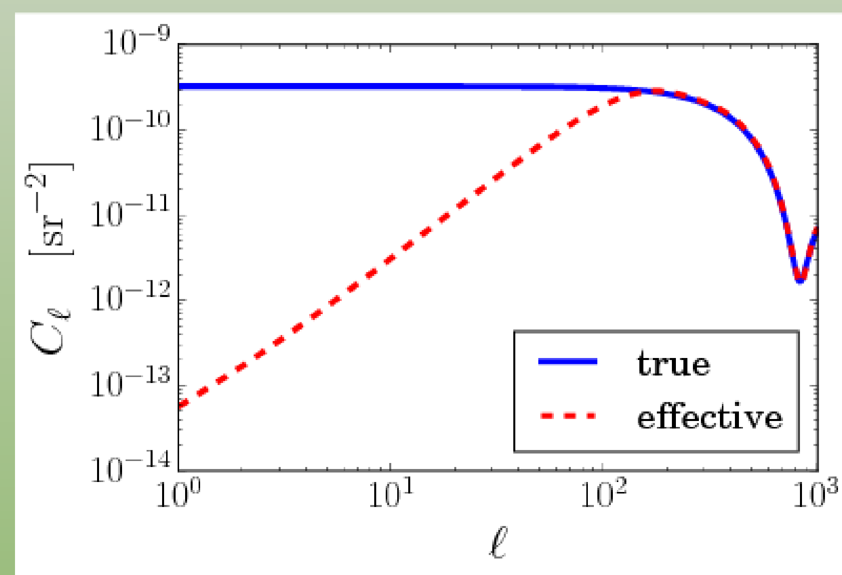
Ratio jackknife/true
auto-redshift

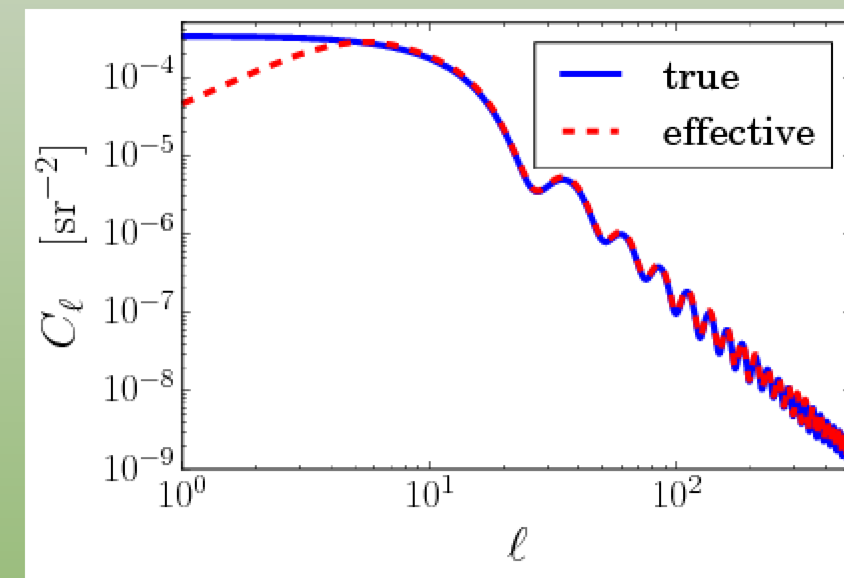


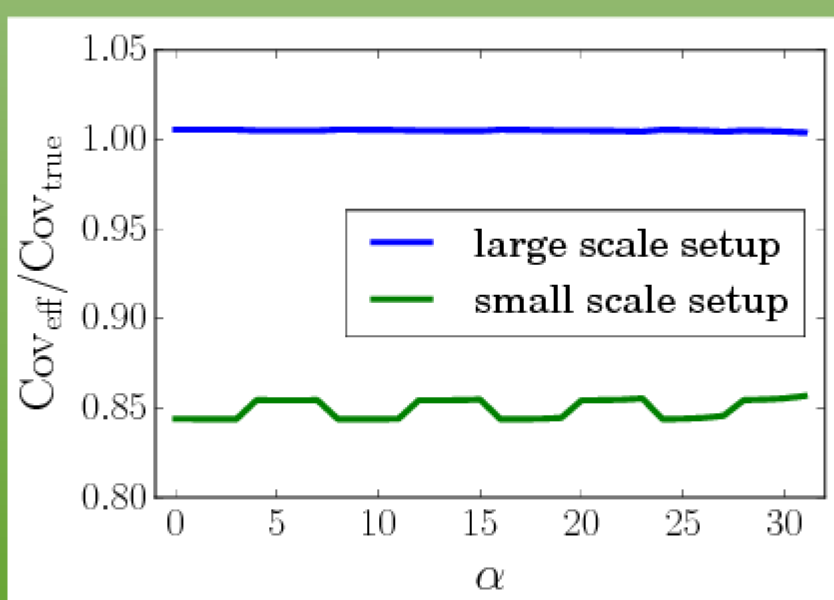
Ratio jackknife/true
cross-redshift

Estimation from a single simulation

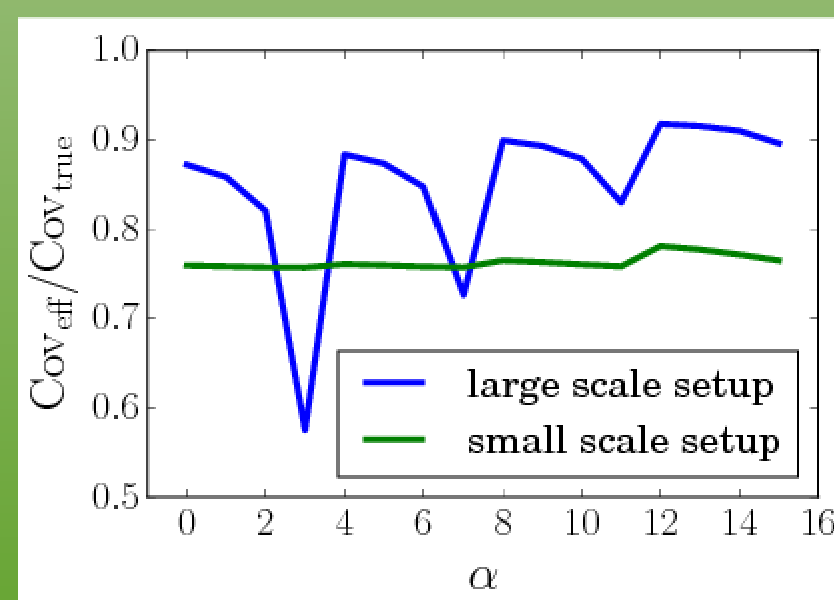








Ratio subsampling/true
auto-redshift



Ratio subsampling/true
cross-redshift

Lacasa & Kunz
arXiv:1703.03337

Other probes

$$\mathrm{Cov}_{\mathrm{SSC}}(\mathcal{O}_1,\mathcal{O}_2) = \int \mathrm{d}V_{12}\,\frac{\partial \mathfrak{o}_1}{\partial\delta_b}\,\frac{\partial \mathfrak{o}_2}{\partial\delta_b}\,\sigma^2(z_1,z_2)$$

Reaction of halo counts to change of background density :

$$\frac{\partial n_h}{\partial\delta_b} = n_h \times b_1$$

Lacasa & Rosenfeld 2016

Reaction of the galaxy power spectrum :

$$\frac{\partial P_{\mathrm{gal}}(k|z)}{\partial\delta_b} = \Big(\underbrace{\frac{68}{21}\,b_1^2}_{\text{second-order perturbation theory}} + \underbrace{2b_1\,b_2}_{\text{second-order Halo bias}}\Big)\,P(k|z) + \underbrace{b_1^{\mathrm{eff-pairs}}(k,z)}_{\text{Halo model (HOD)}}/\overline{n}_{\mathrm{gal}}(z) + \underbrace{b_1/\overline{n}_{\mathrm{gal}}(z)}_{\text{shot-noise}}$$

- misses dilation effect from Li, Hu & Takada (2014) ?
- weak-lensing : similar except with matter power spectrum, bias etc instead of galaxies

SSC and the galaxy power spectrum

Saturation of the information content at trans-linear scales (though fully non-linear scales can recover information) : Rimes & Hamilton (2005, 2006), Neyrinck et al. (2006, 2007), Carron et al. (2015)

$$\mathrm{Cov}\left(\mathbf{X},\mathbf{X}\right) = \begin{pmatrix} \mathrm{Cov}\left(C_\ell^{\mathrm{gal}}, C_\ell^{\mathrm{gal}}\right) & \mathrm{Cov}\left(C_\ell^{\mathrm{gal}}, N_{\mathrm{cl}}\right) \\ \mathrm{Cov}\left(N_{\mathrm{cl}}, C_\ell^{\mathrm{gal}}\right) & \mathrm{Cov}\left(N_{\mathrm{cl}}, N_{\mathrm{cl}}\right) \end{pmatrix}$$

- 9 redshift bins (hence 9 subplots)
  z=0.1-1 with Dz=0.1

- 3 mass bins
logM = 14-15.5 with DlogM=0.5

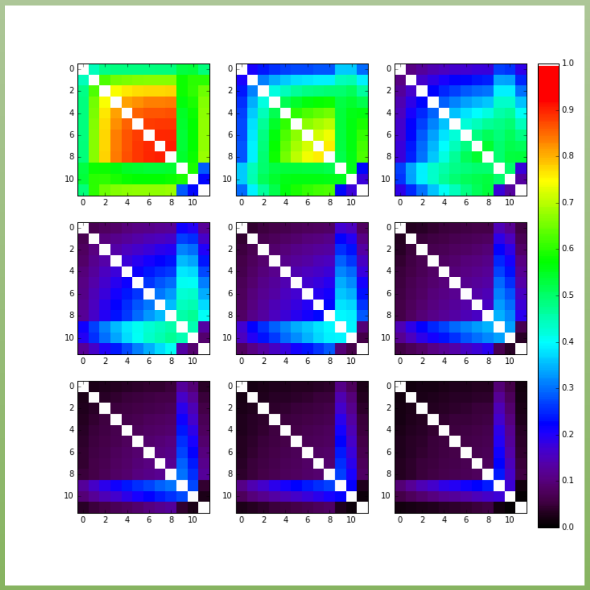
- 9 multipole bins
ell=30-300 Dl=30

- Standard = Gaussian for Cl,
  Poissonian for Ncl

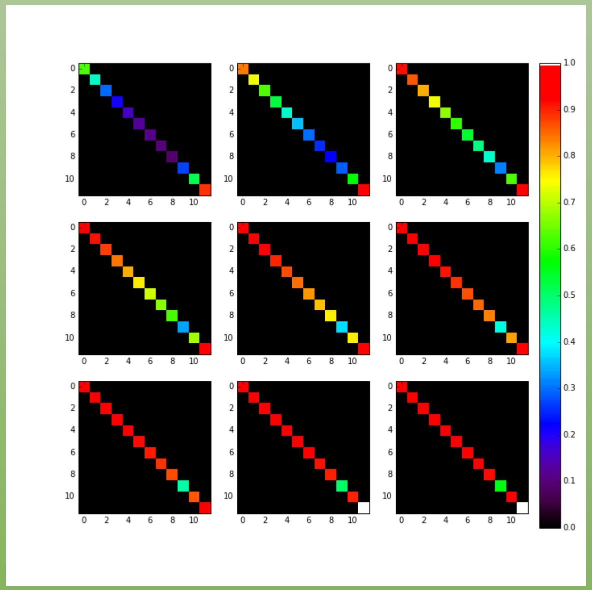
- Correlation matrices (normalised to
  1=white on the diagonal)

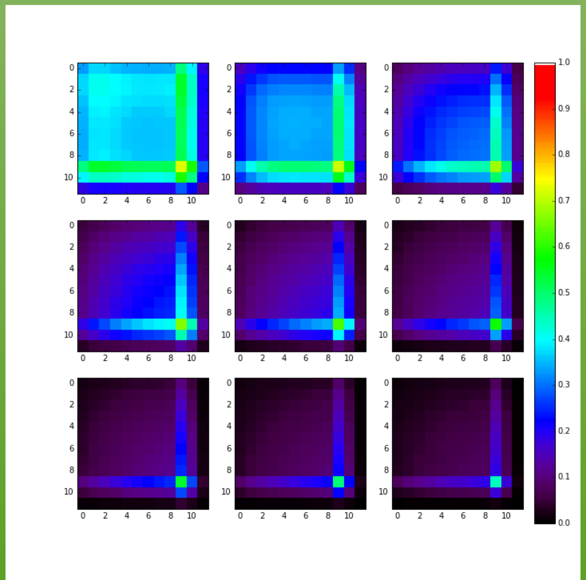
Lacasa & Rosenfeld (2016)

total

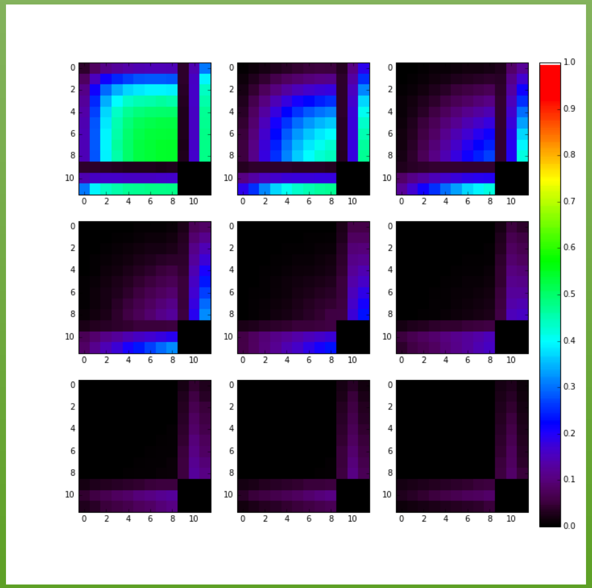


standard





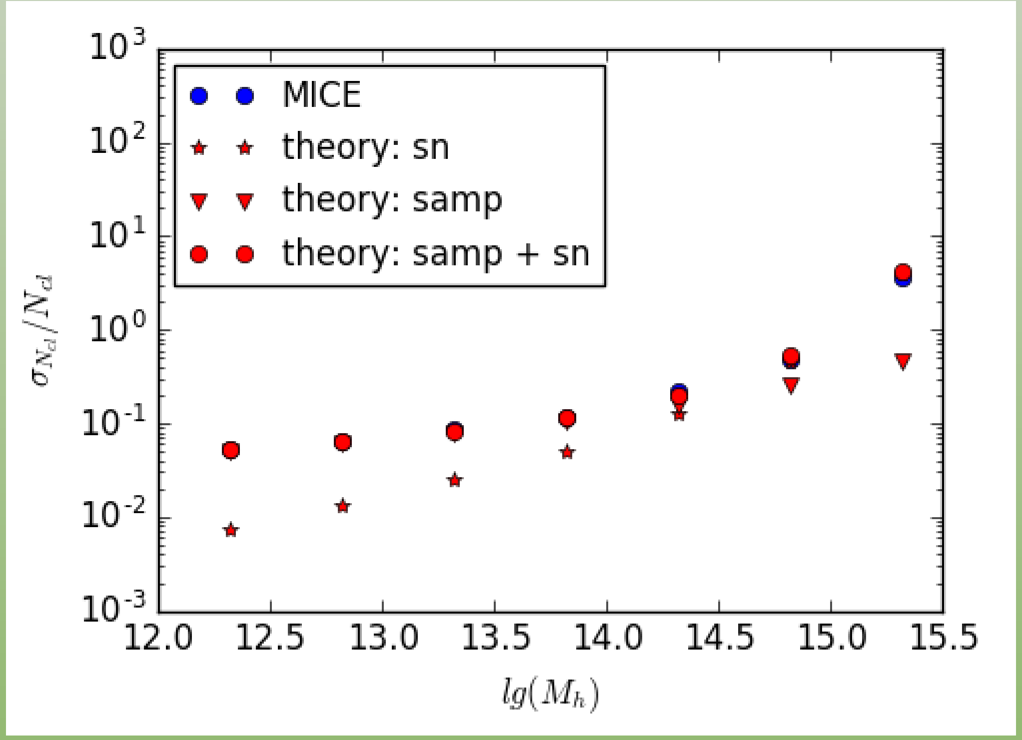
SSC



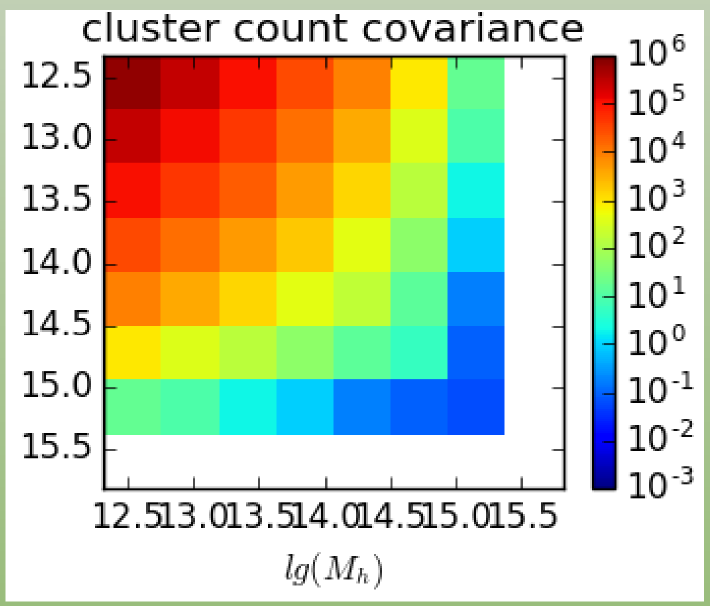
other NG

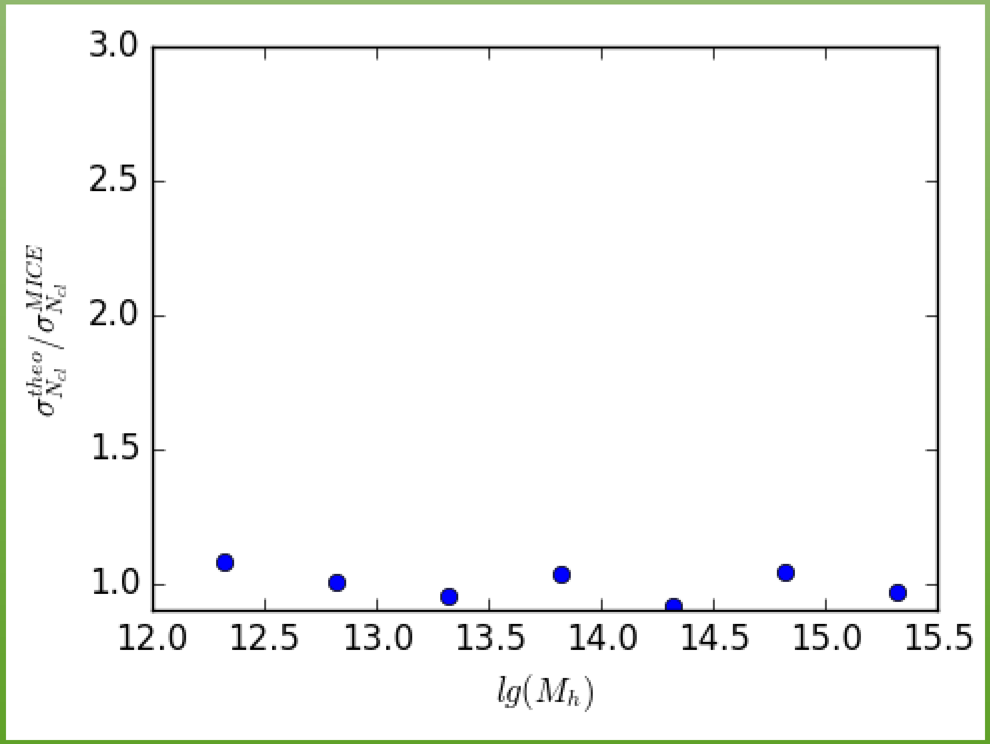
Comparison with MICE I : Cluster counts

Error bars comparison

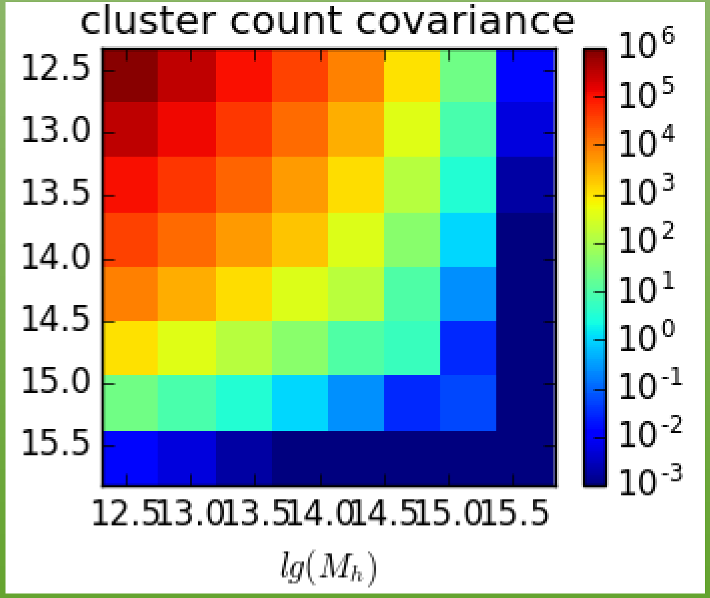


Measured covariance matrix





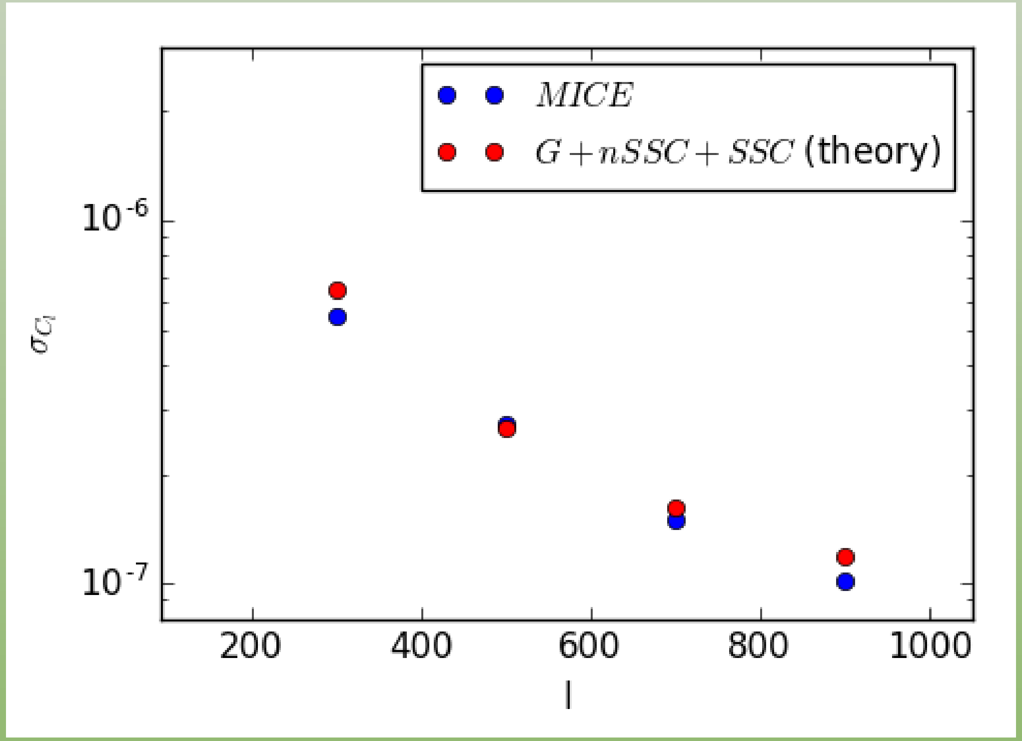
Error bars ratio

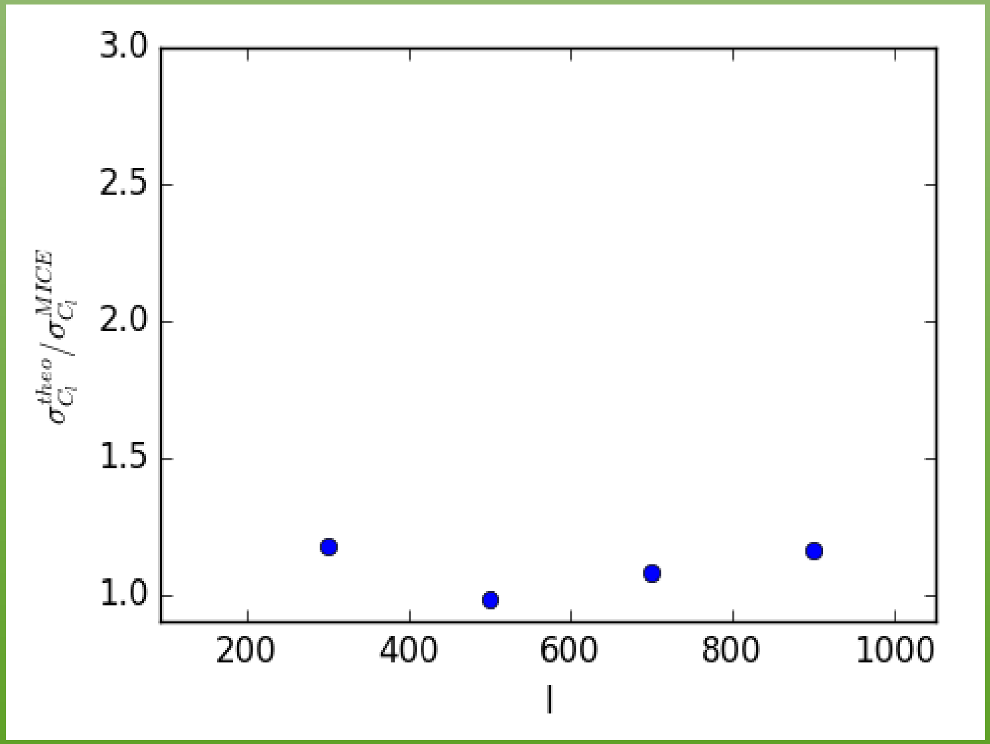


Theoretical covariance matrix

Comparison with MICE II : galaxy C(l)

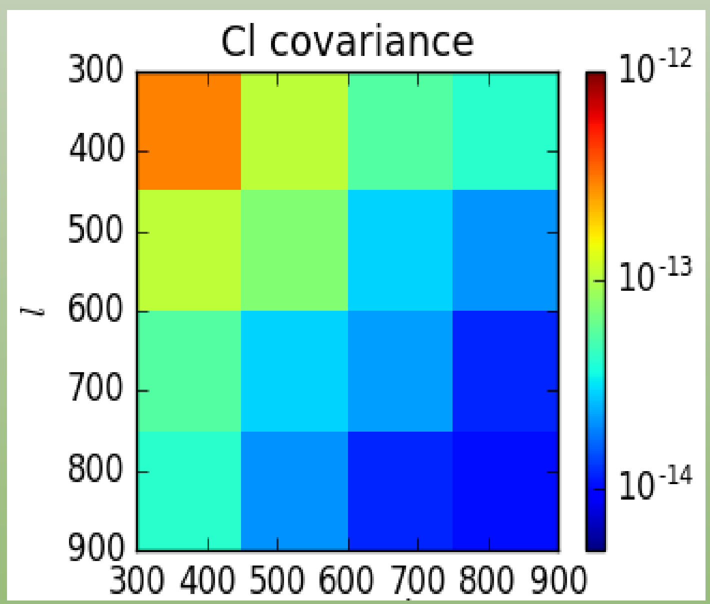
Error bars comparison

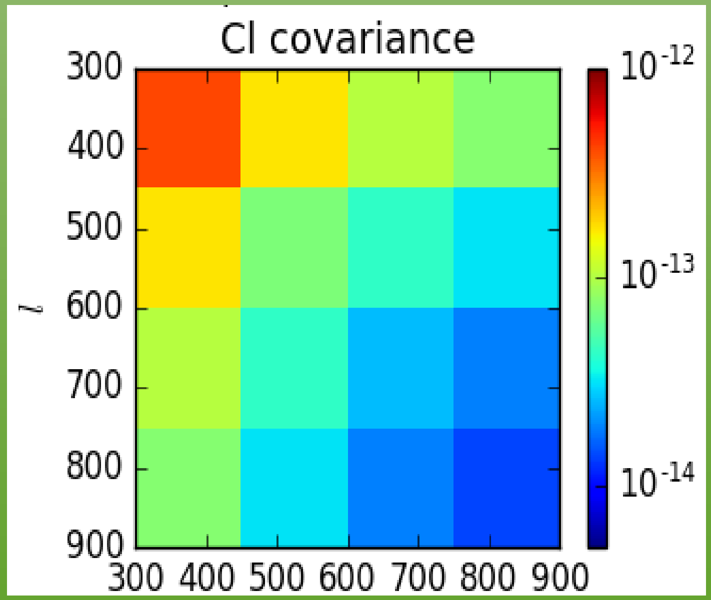




Error bars ratio

Measured covariance matrix

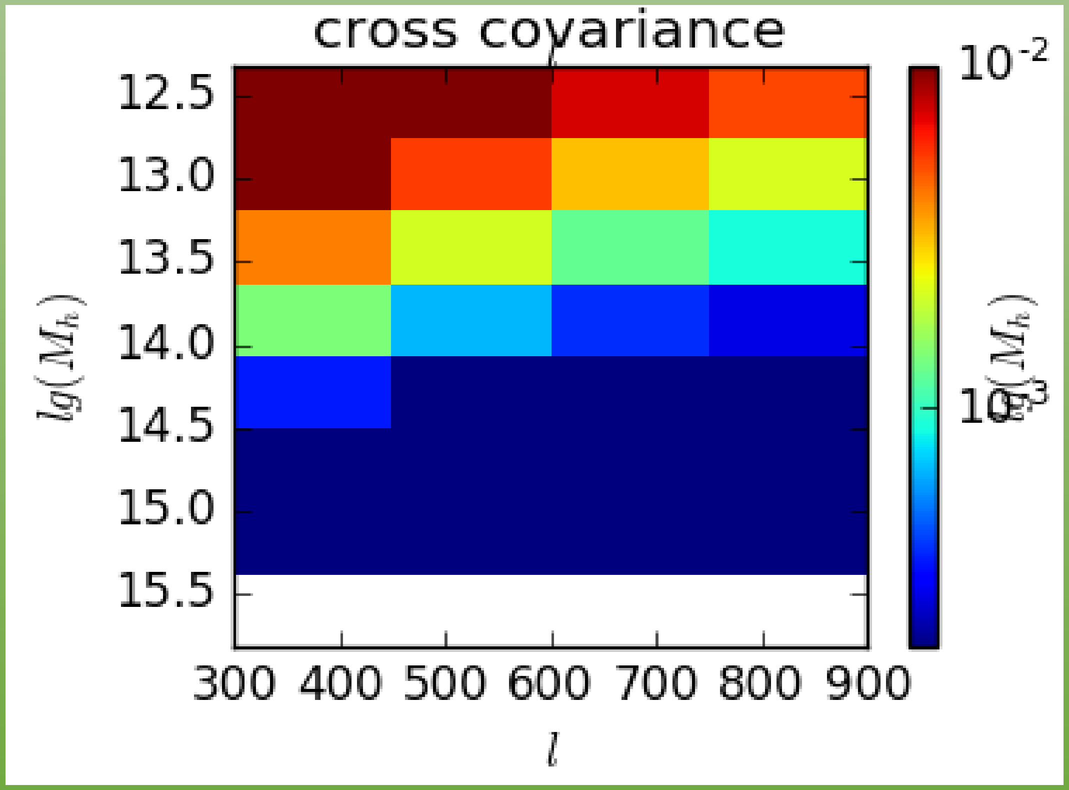




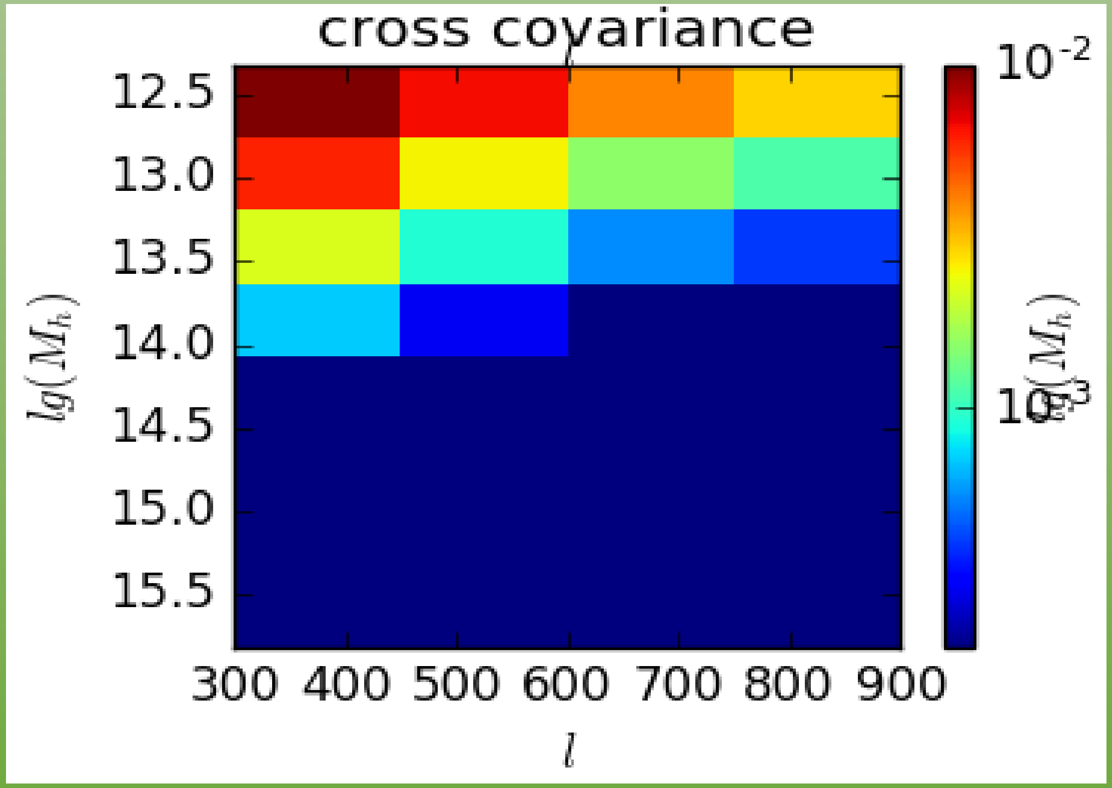
Theoretical covariance matrix

Comparison with MICE III : cross-covariance

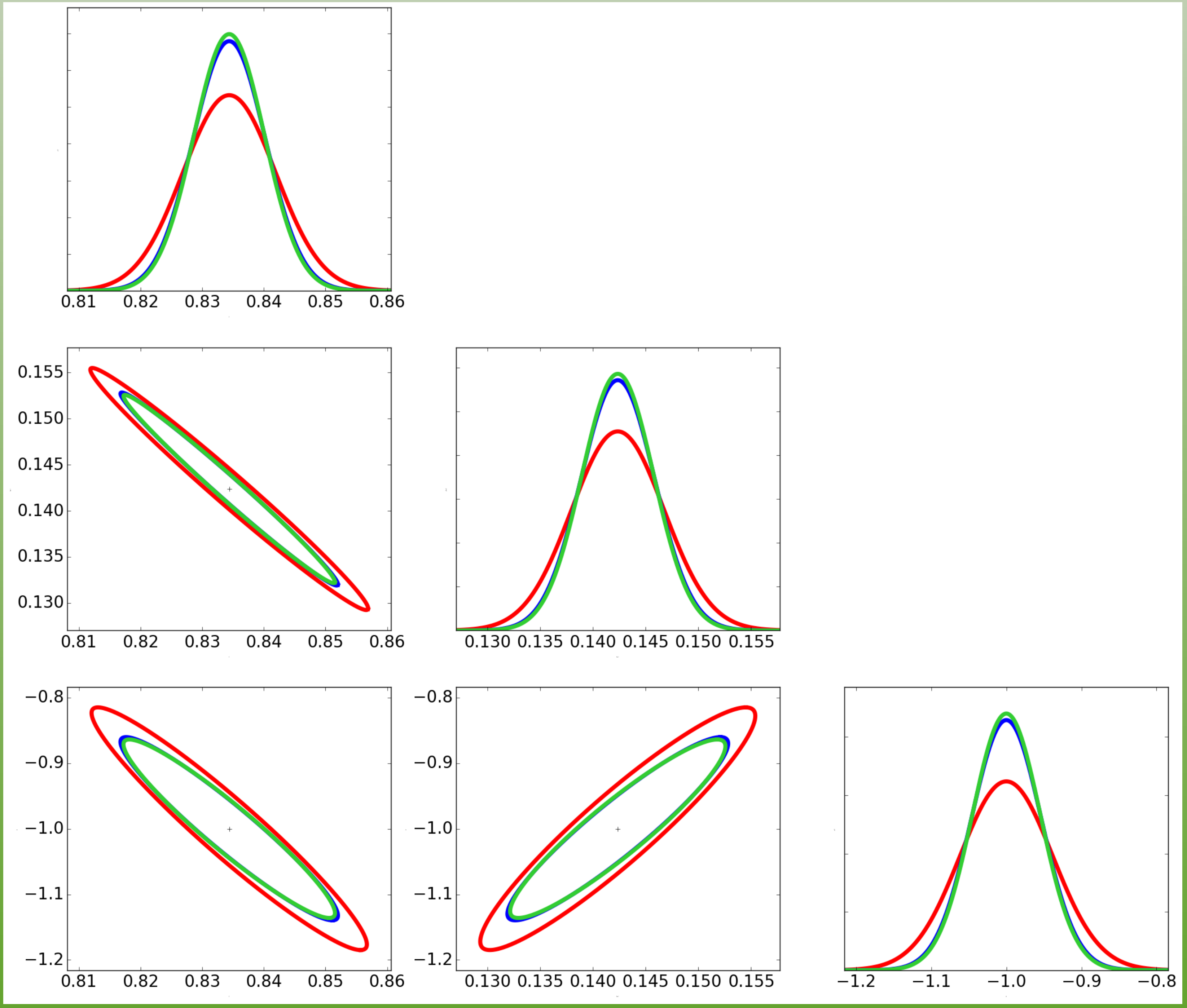
Measured covariance matrix



Theoretical covariance matrix



Impact on cosmological parameters



σ_8 $\Omega_m h^2$ w

standard terms

NG no SSC

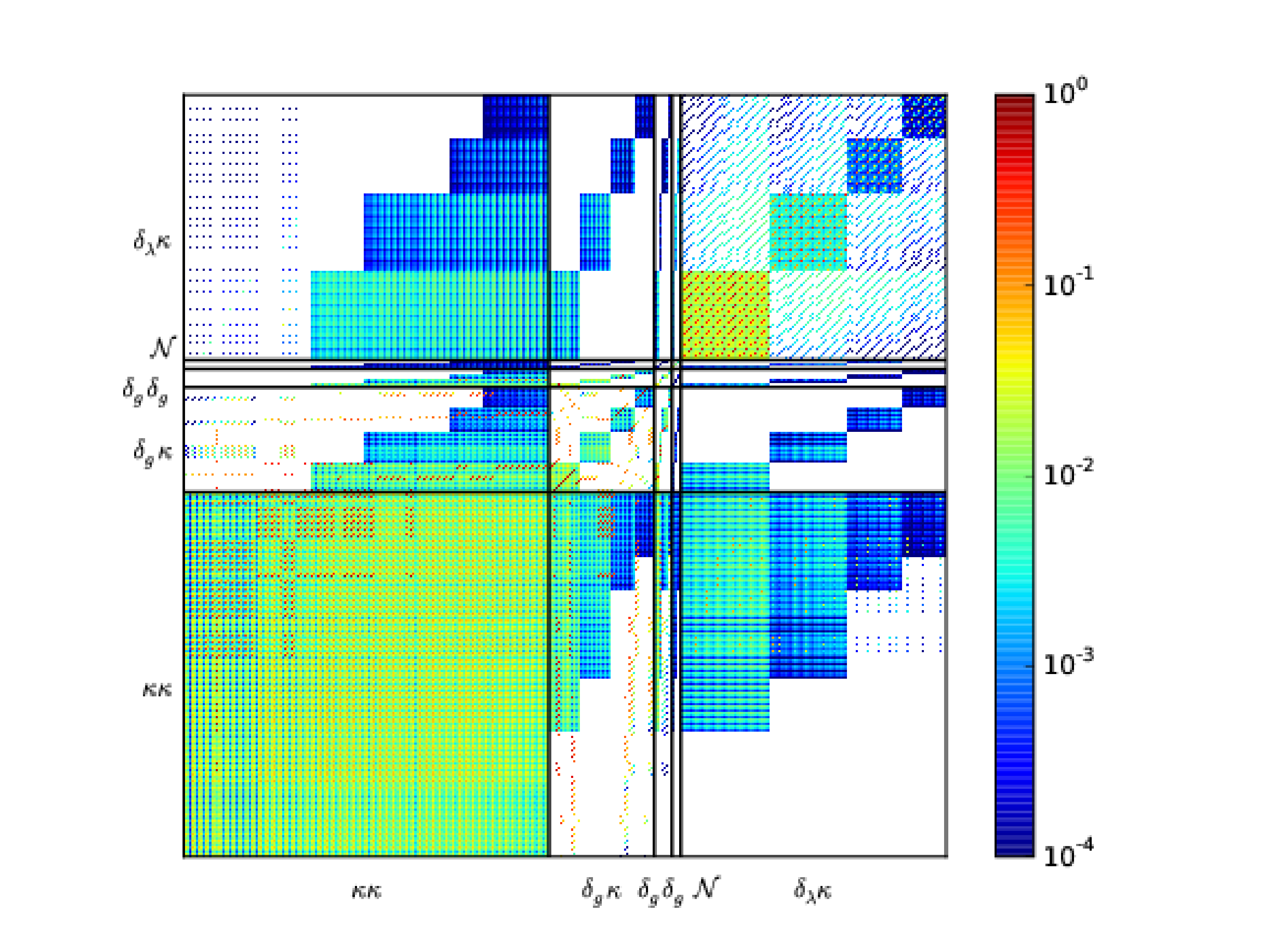
total

Error bar increase

σ_8 : 31%
$\Omega_m h^2$: 29%
w : 36%

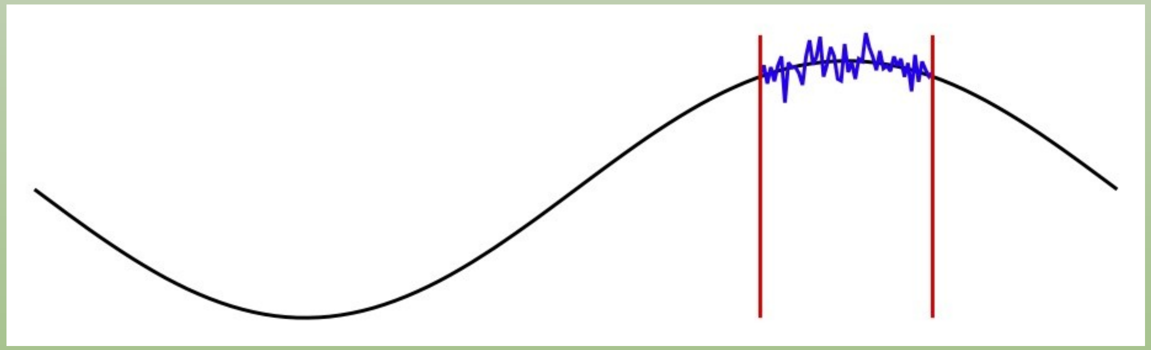
FoM increases by a factor 2.2

Combining probes



Krause & Eifler 2016

Equivalence principle and SSC



Average gravitationnal potential and its gradient have no effect on observables

- Consistency relations of Large Scale Structure : Creminelli et al. 2013, 2014 (+ many later)
- Application to covariance of the matter power spectrum : Barreira & Schmidt arXiv:1703.09212

- Separate universe simulations :
  region with $\delta_b \neq 0$ can be simulated as an independent universe with different cosmological parameters (different Ω_m, H_0, presence of a curvature)
- These simulations are used to calibrate : response of observables to background change
  Li, Hu & Takada (2014, 2016), Wagner et al. (2015), Paranjape & Padmanabhan(2016) ...

Separate universe can be taken analytically (e.g. Nambu 2003, Rigopoulos & Shellard 2005)
may mean that we should be able to calibrate SSC non-perturbatively with making predictions of observables with different cosmologies (including curvature)
=> range of viable SSC prediction = range of viable observable prediction

Beyond SSC

- SSC alone gives near 100% correlation : it's a coherent change of shape (for the mass function or power spectrum)

=> erases information on the amplitude but not on the slope
=> some cosmological parameters are affected (Ω_m, σ_8, H_0)
but others dont care (n_S, f_{NL}, neutrinos, WDM, SIDM)

- Normalising by the actual number of galaxies partially cancel one SSC term for the galaxy power spectrum
  But :
  - joint analysis Ngal – power spectrum would be better
  - not possible for weak-lensing

- Combination with cluster counts mitigates SSC both for weak-lensing (Takada & Bridle 2007, Takada & Spergel 2014) and galaxy clustering (Lacasa & Rosenfeld 2016)

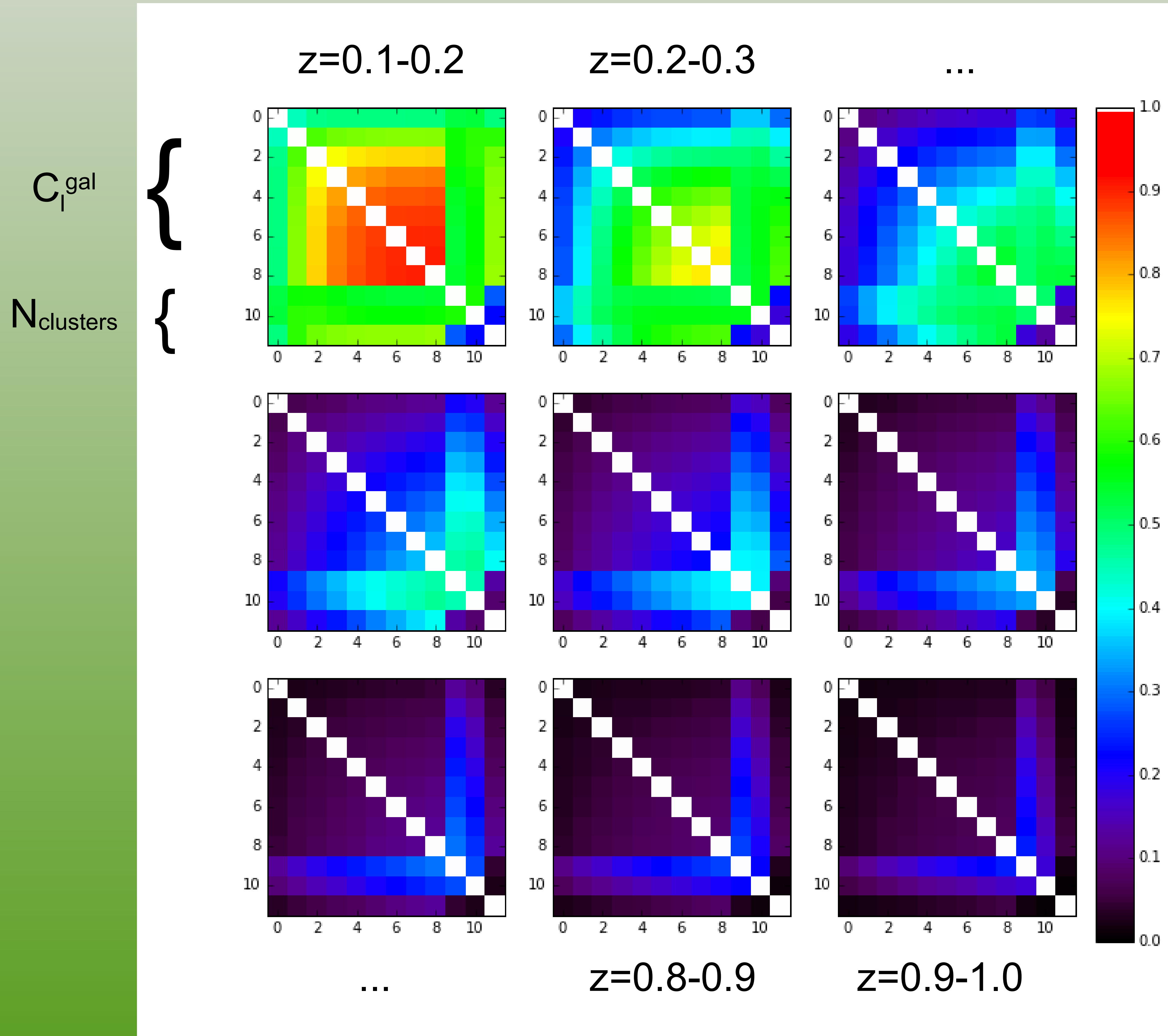
- Power of extra statistics :
  - 1-point probes (galaxy counts, cluster counts, shear peaks)
  - lognormal model (Carron & Szapudi 2015)

Conclusions

- SSC is a source of covariance from long wavelength modes larger than the survey
- Dominates high signal-to-noise regime (low mass, small angular scales) within reach of current and future galaxy surveys
- Poorly estimated from data itself or classical N-body simulation
- Analytical modelisation possible but needs to be careful
- Still open exciting theoretical questions (consistency relation of LSS, separate universe, equivalence principle...)
- Calls for probe combination, including 1-point statistics
- Using information in the non-linear regime (e.g. with halo model) can help a lot !

Thanks for the attention

Joint covariance



Correlation matrix : $C_{ij}/\mathrm{sqrt}(C_{ii}*C_{jj})$

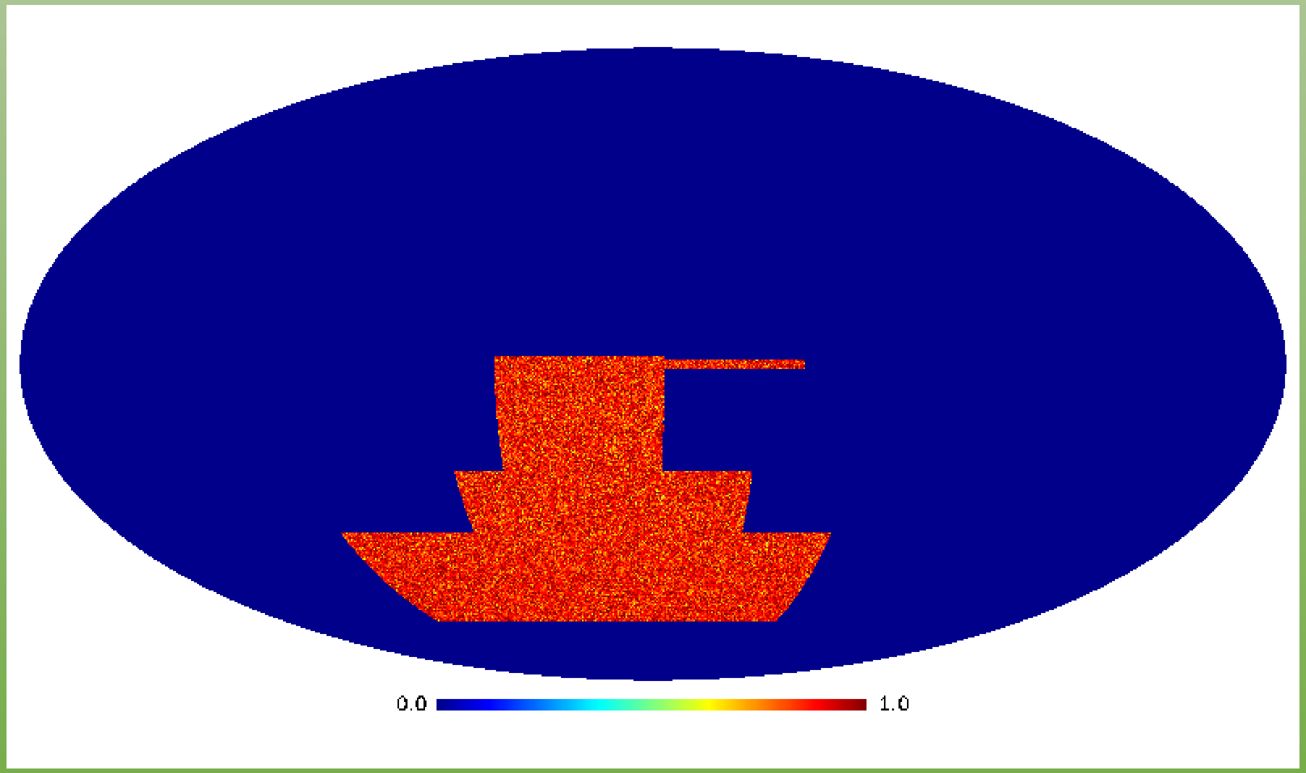
Cross-covariance is important at all redshifts.

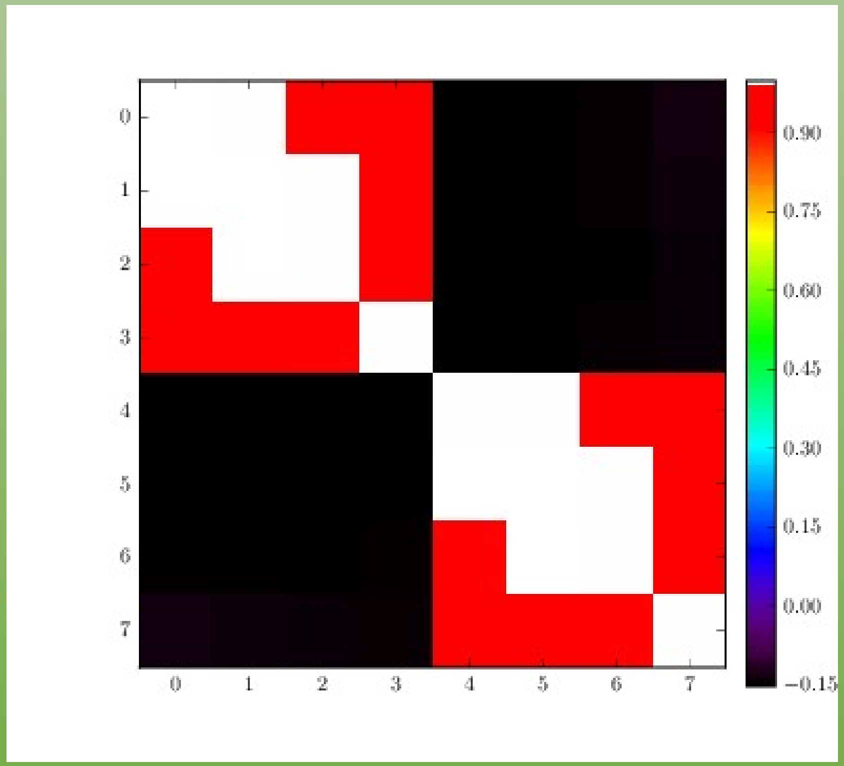
Particularly for the smaller angular scales

Lacasa & Rosenfeld 2016
arXiv:1603.00918

Analytical SSC predictions...

… are possible, and can account for an arbitrary survey geometry



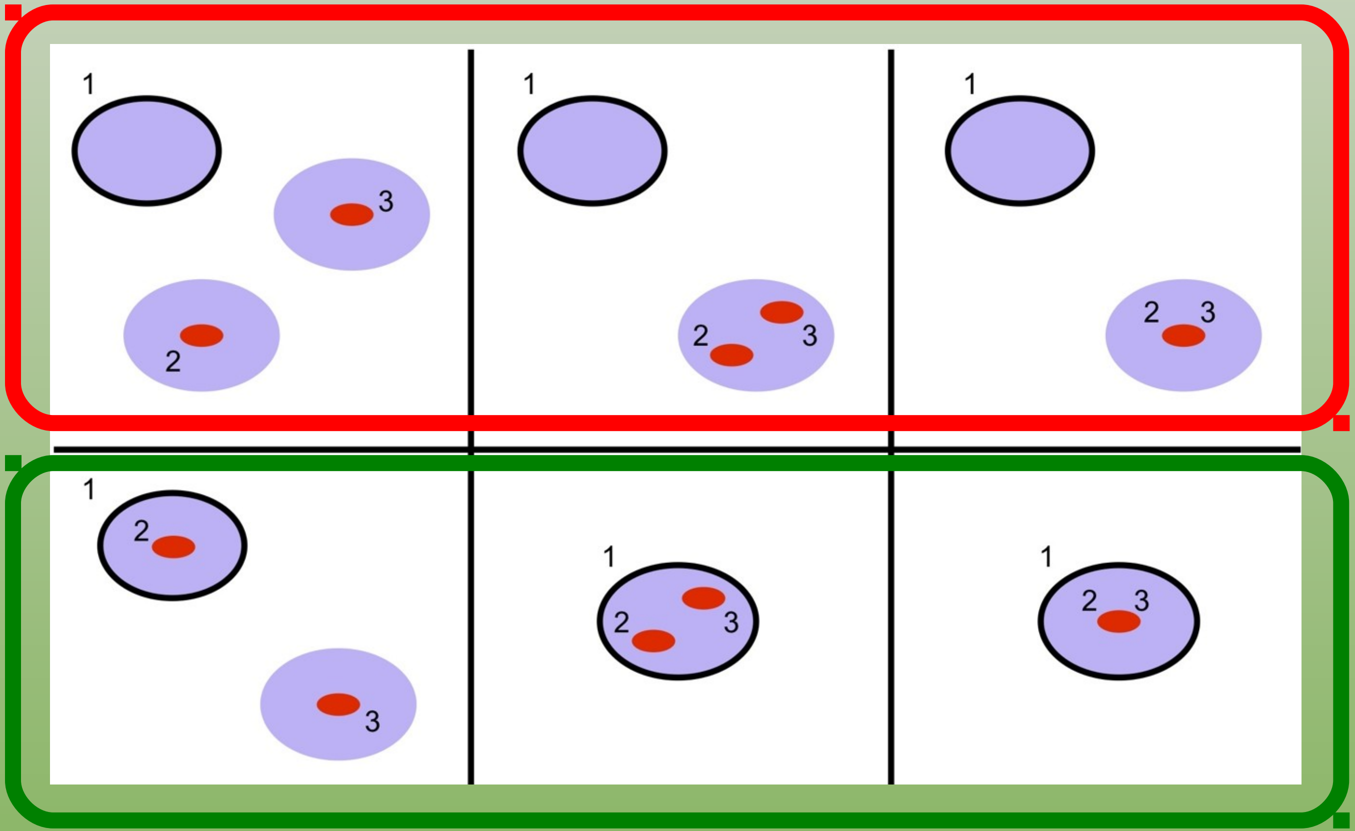


Cross-z covariance is important for large surveys

Lacasa, Lima & Aguena 2016
arXiv:1612.05958

Thanks for the attention

Diagrammatic



Super-sample covariance (SSC)

How observables fluctuate together as they are modulated by large scale structure.

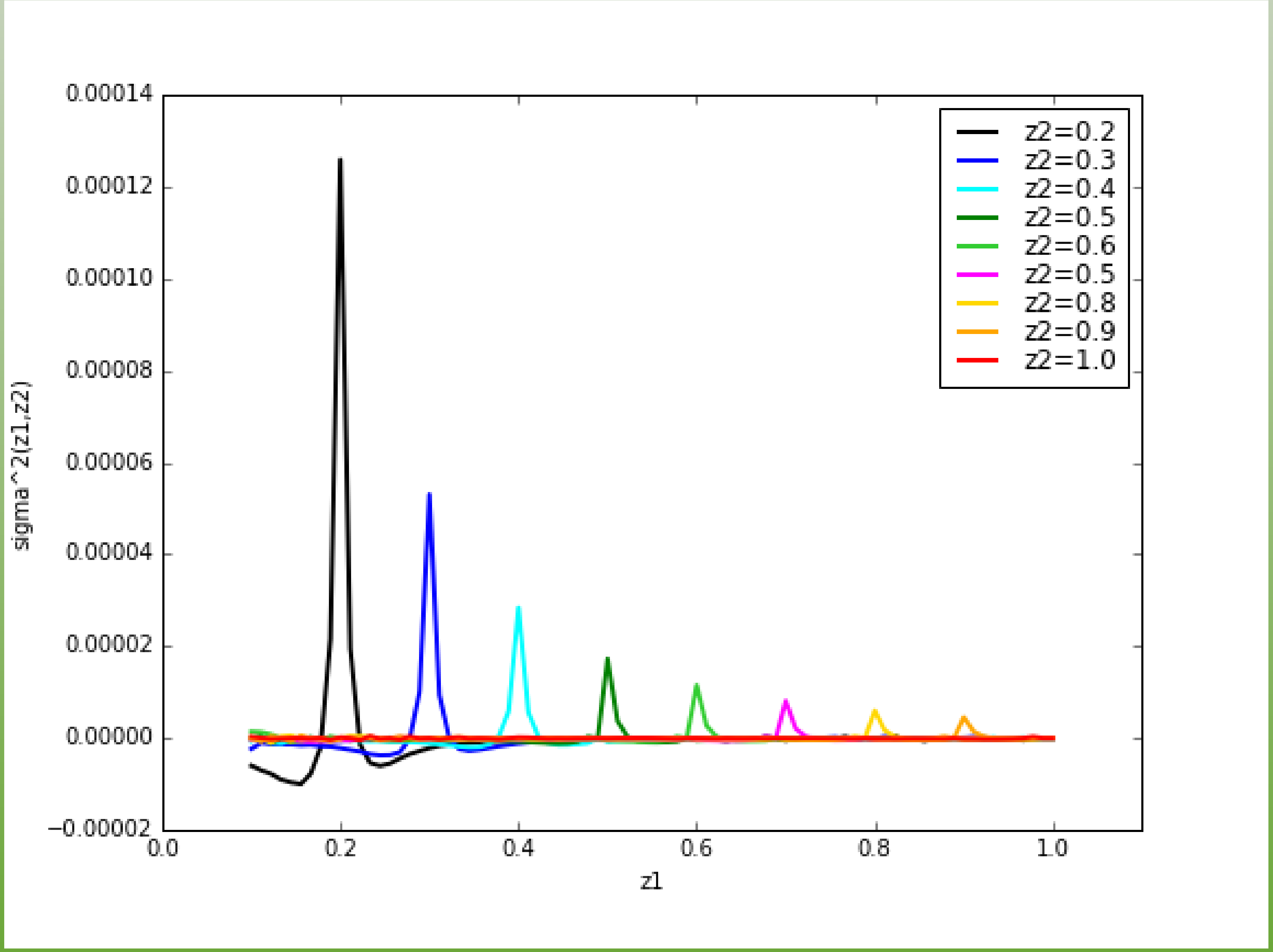
How the clusters source the galaxy power spectrum

Sensitive to the Halo Occupation Distribution.

Each diagram → a term of the halo-galaxy-galaxy 3-point function
→ a term of the cross-covariance

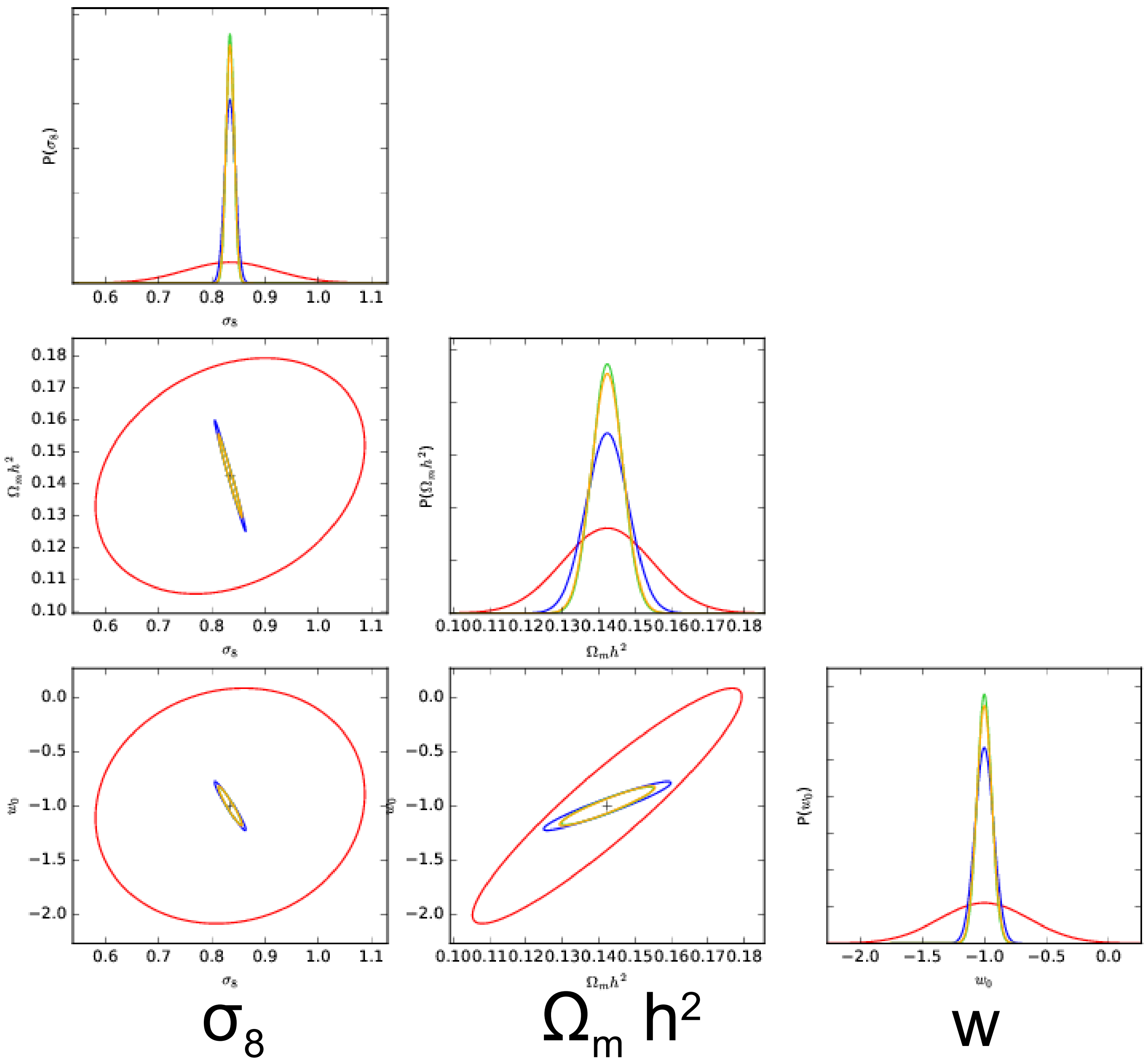
3-halo term splits into contributions from
- perturbation theory (2PT)
- non-linear halo bias (b_2)

$\sigma^2(z_1,z_2)$



Covariance of the matter average density in the redshift shells z_1 and z_2

Cosmological case



C(l) constraints

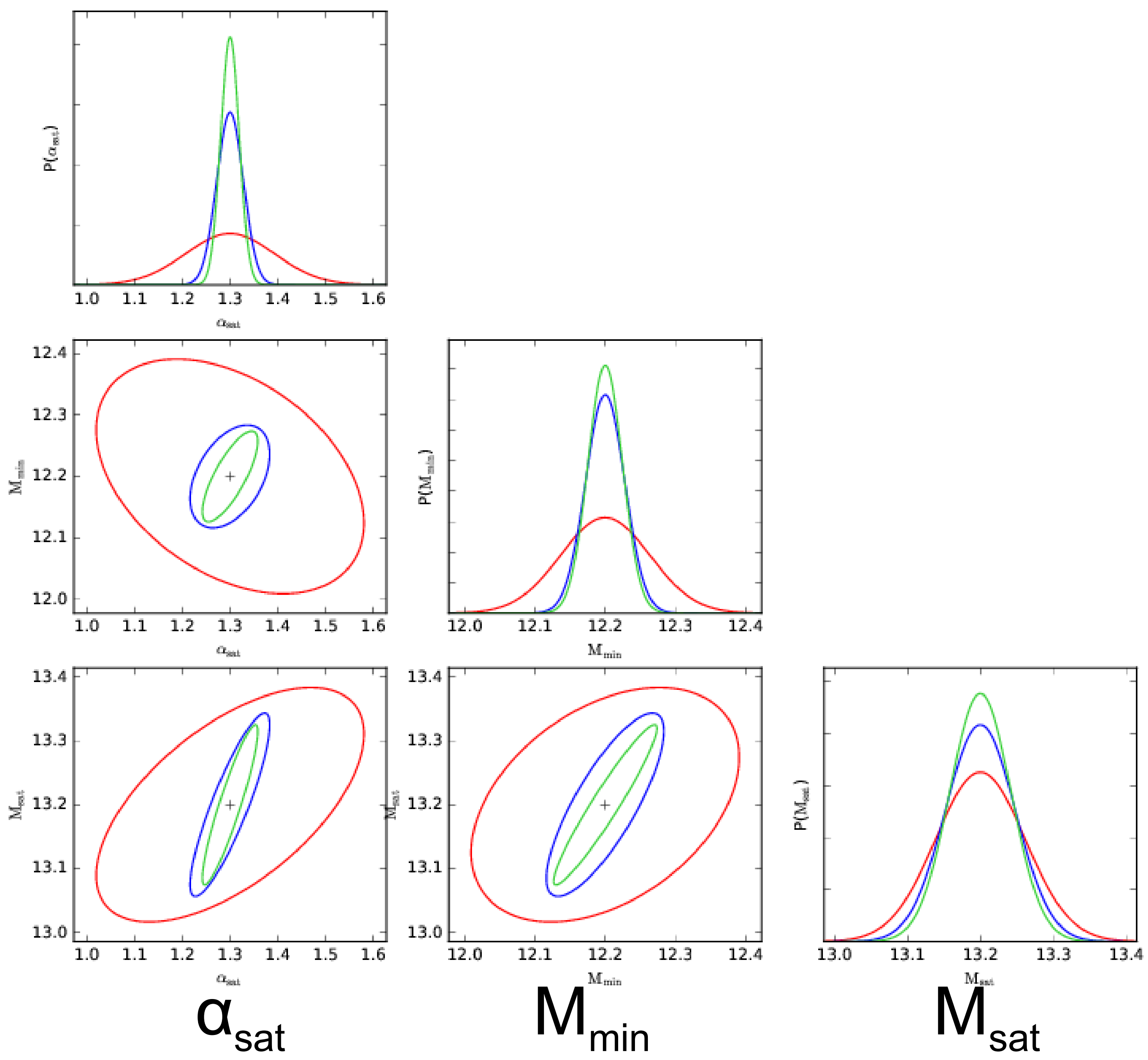
Ncl constraints

Joint without cross-cov
= naive

Joint with cross-cov
= realist

small difference with or without cross-cov

HOD case



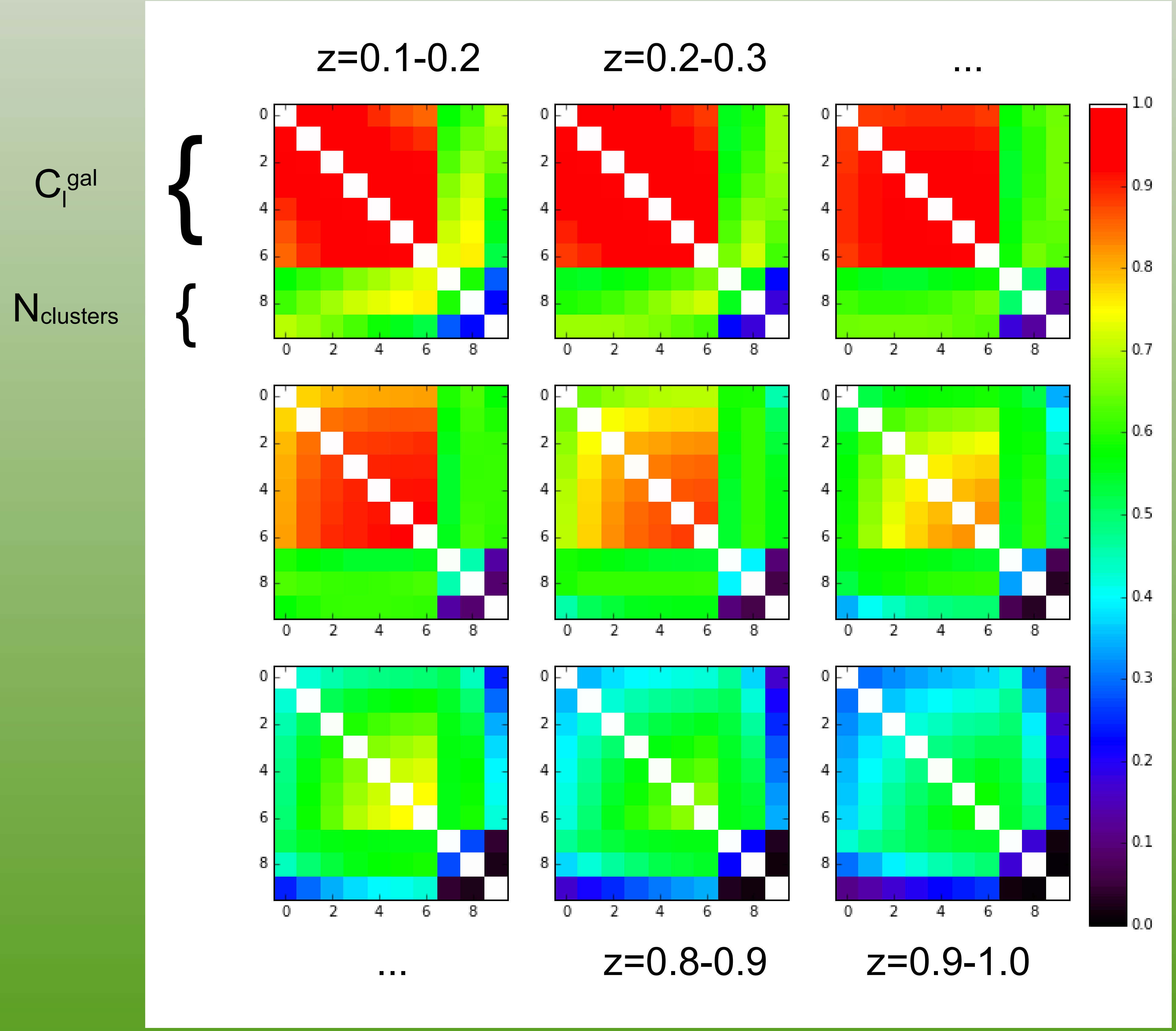
C(l) constraints

Joint without cross-cov
= naive

Joint with cross-cov
= realist

Better with than without

On smaller angular scales



Diagrams for the galaxy trispectrum