Analytical Tools for Future LSS Surveys

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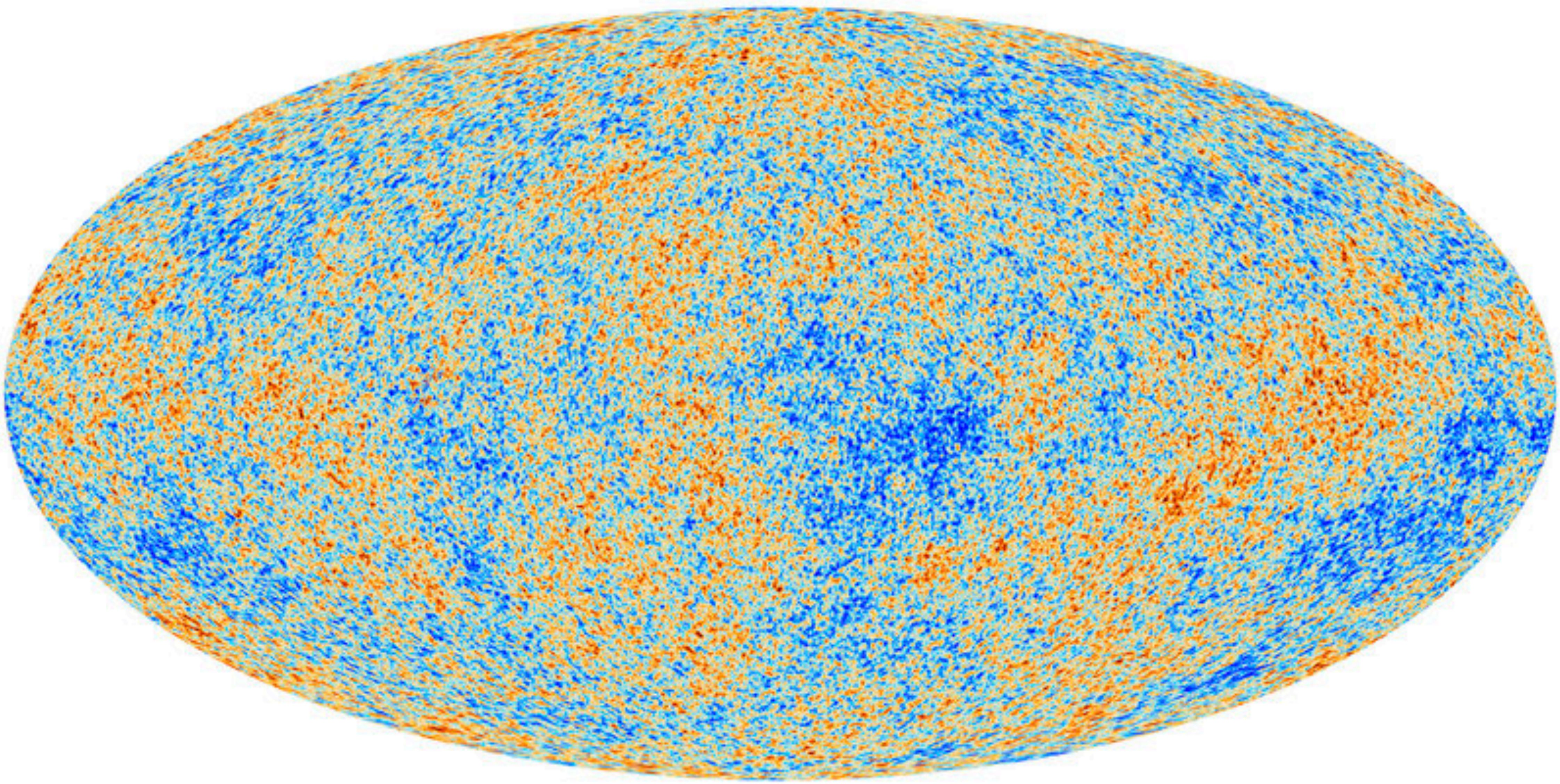
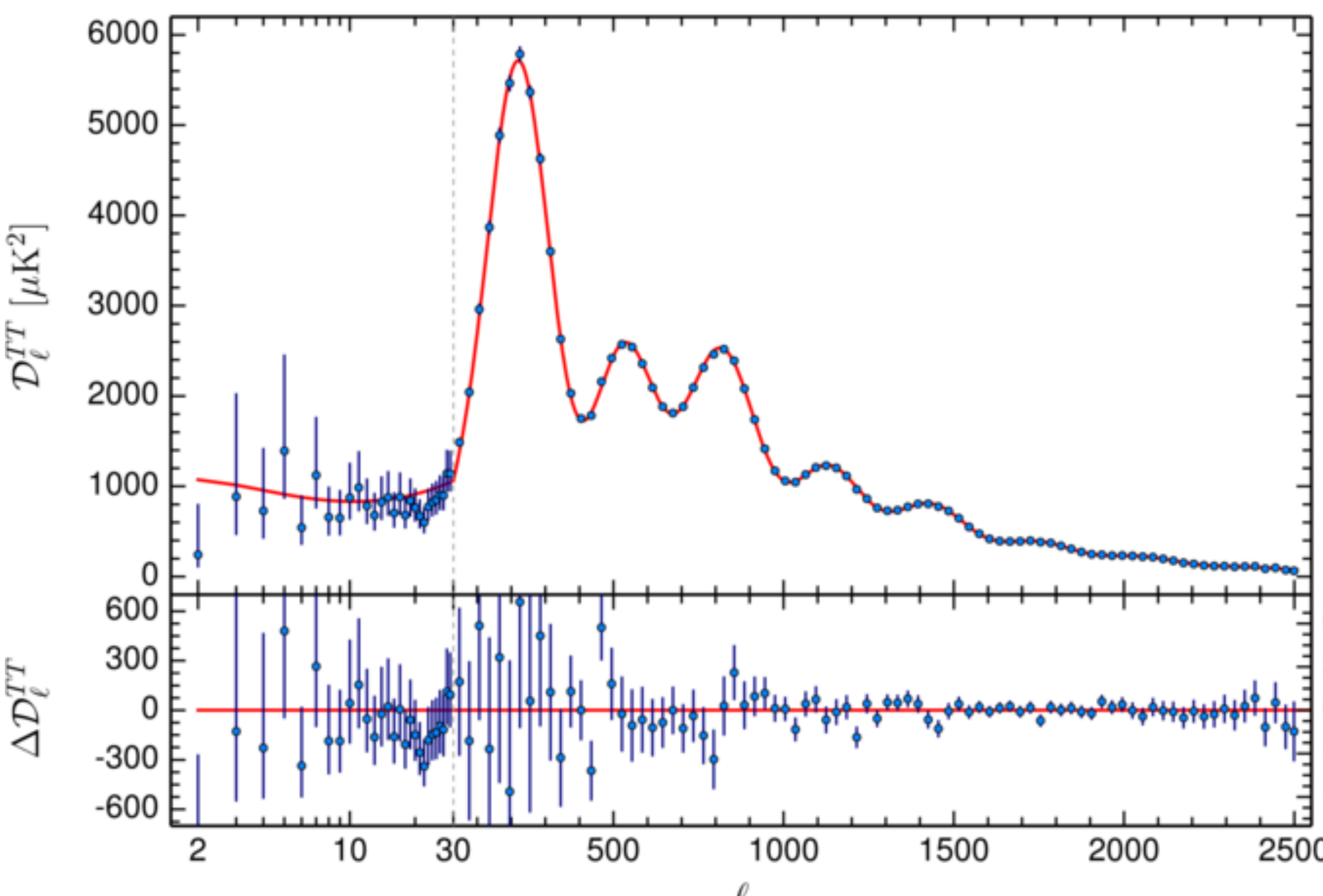
ΛCDM very successful in describing the universe



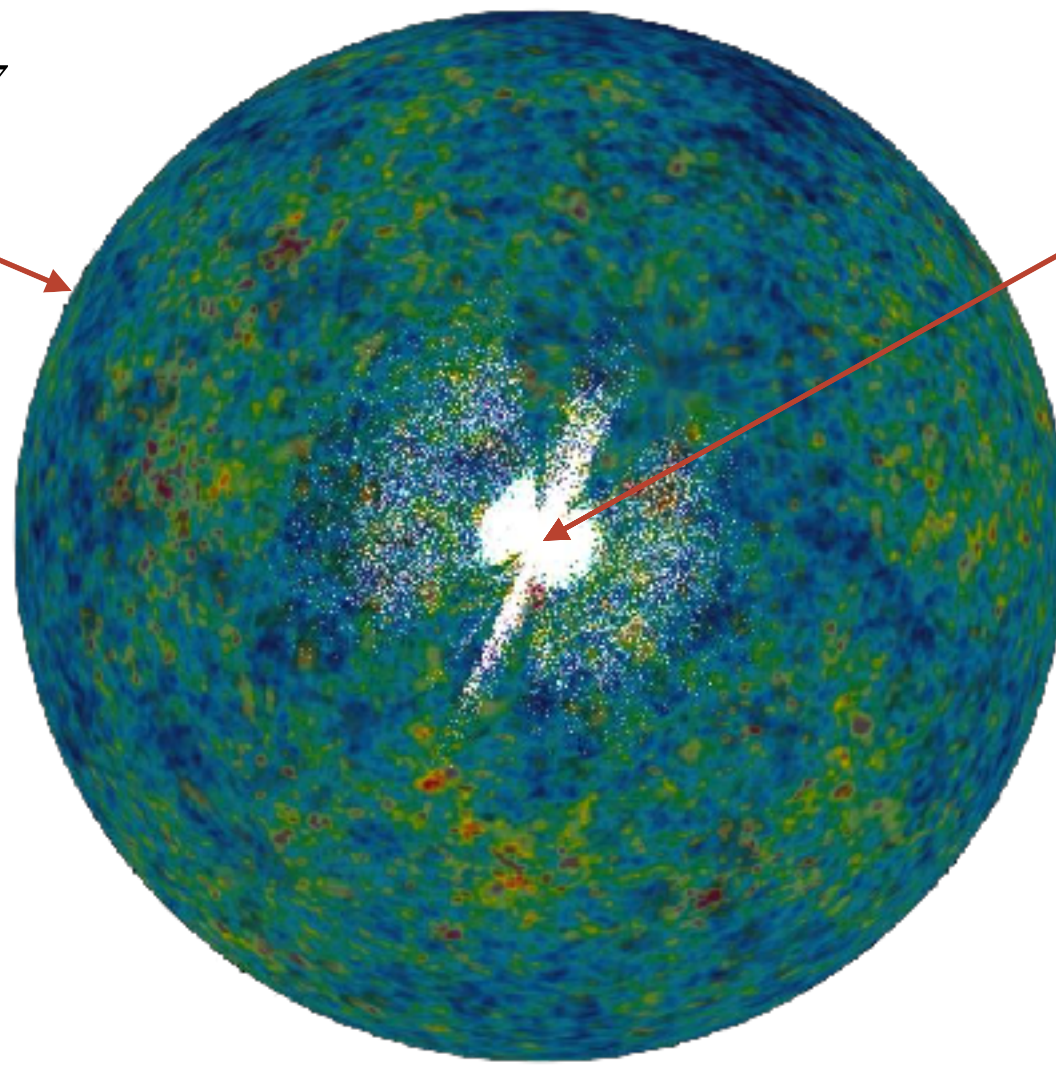
The fluctuations are characterised by the power spectrum

ΛCDM very successful in describing the universe



Can we go beyond ΛCDM using LSS?

$N_{\rm CMB} \approx l_{\rm max}^2 \sim 10^7$



$N_{\rm LSS} \approx \left(\frac{k_{\rm max}}{k_{\rm min}}\right)^3$

Galaxies

Lyα

21cm

…

Can we go beyond ΛCDM using LSS?

Many ongoing and future LSS surveys aim at:

Tighter constraints on cosmological parameters

Measurements of neutrino masses, non-Gaussianities, running…

Possible surprises in the dark sector, modified gravity…

The main challenge is the nonlinear evolution

Analytical tools are essential for the modelling of LSS
and data analysis

Outline

Review of perturbation theory for DM and biased tracers

Modelling of the BAO peak — IR resummation

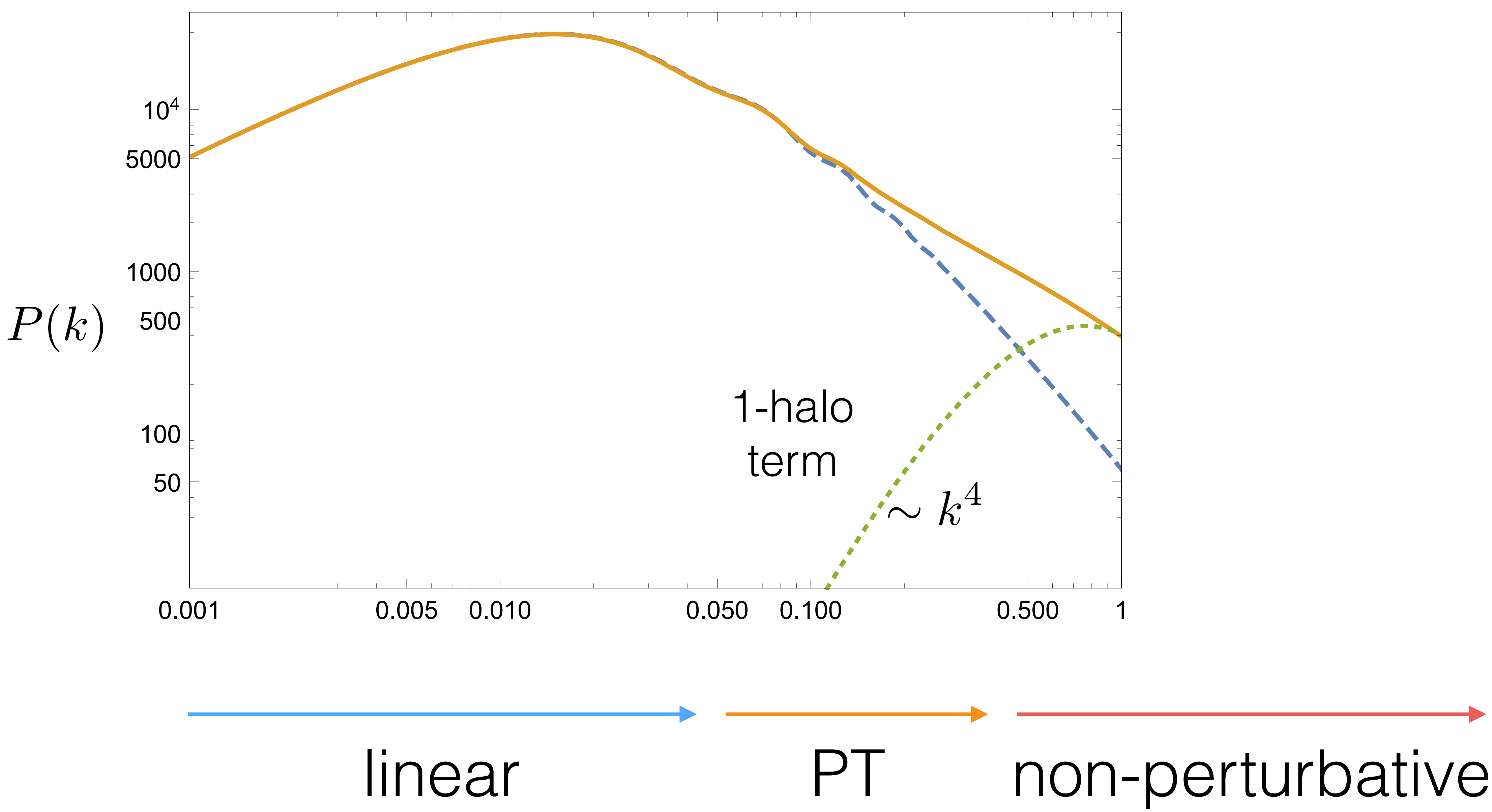
Some open questions and future directions

Part I

Perturbative approach to LSS

PT approach to LSS

The goal is to provide consistent templates for all observables on large scales (all n-point functions, BAO peak/wiggles…)



PT approach to LSS

Matter behaves as a fluid on large scales

On large scales the density fluctuations are small

$$\left.\begin{aligned} \partial_\tau \delta + \nabla[(1+\delta)\boldsymbol{v}] &= 0 \\ \partial_\tau \boldsymbol{v} + \mathcal{H}\boldsymbol{v} + \nabla\Phi + \boldsymbol{v}\cdot\nabla\boldsymbol{v} &= \cdots \\ \nabla^2\Phi &= \frac{3}{2}\mathcal{H}^2\Omega_m\delta \end{aligned}\right\}$$

SPT equations

Scoccimarro, Frieman (1996)

Bernardeu, Colombi, Scoccimarro (2002)

One can find perturbative solutions

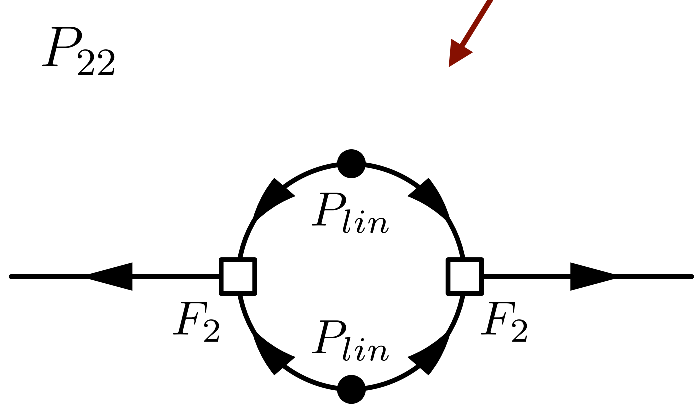
$$\delta^{(n)}(\boldsymbol{k}) = \int_{\boldsymbol{q}_i} F_n(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_n)\delta^0(\boldsymbol{q}_1)\cdots\delta^0(\boldsymbol{q}_n)$$

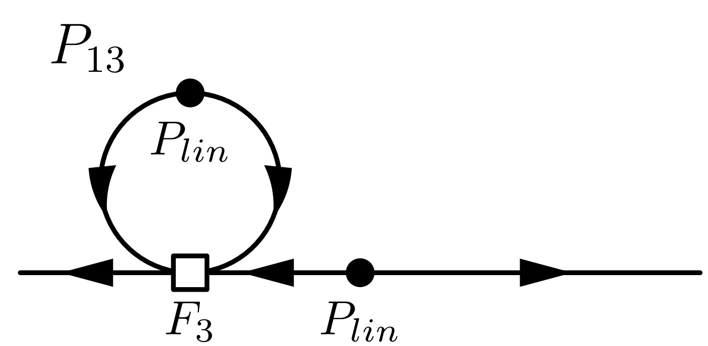
PT approach to LSS

The nonlinear power spectrum

$$P_{\rm NL}(k,\tau) = D^2(\tau)P_{\rm lin}(k) + P_{1-\rm loop}(k,\tau) + \cdots$$

Scoccimarro, Frieman (1996)





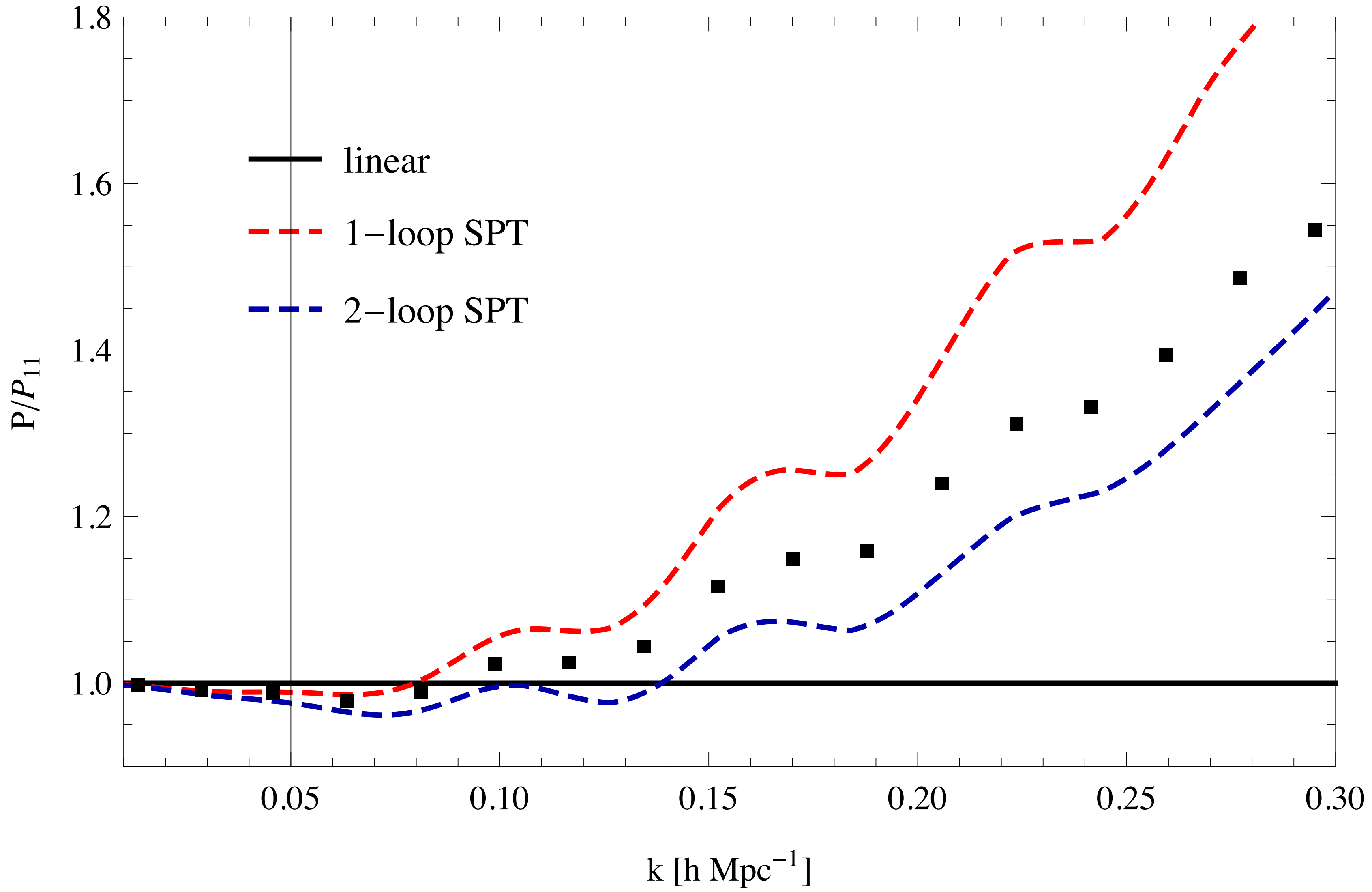
$$P_{22}(k) = 2\int_{\boldsymbol{q}} F_2^2(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})P_{\rm lin}(q)P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$P_{13}(k) = 6P_{\rm lin}(k)\int_{\boldsymbol{q}} F_3(\boldsymbol{q},-\boldsymbol{q},\boldsymbol{k})P_{\rm lin}(q)$$

PT approach to LSS

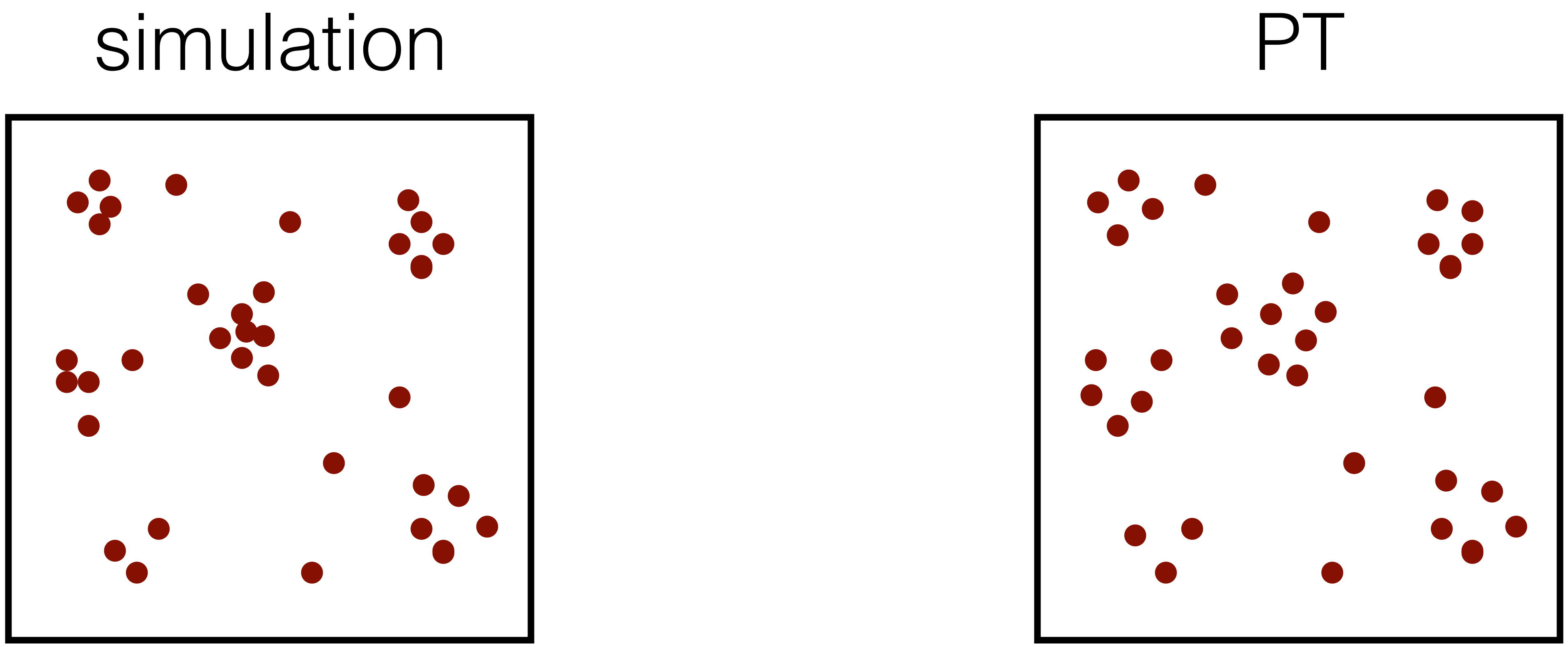
A well known problem, SPT seems not to converge

In power-law cosmologies the loops may even be infinite



PT approach to LSS

Simulations and PT conserve mass and momentum



$$\delta_{\mathrm{NL}}^{\mathrm{sim}}(\boldsymbol{k}) - \delta_{\mathrm{NL}}^{\mathrm{PT}}(\boldsymbol{k}) \sim R^2 k^2 \delta_{\mathrm{lin}}(\boldsymbol{k}) + \cdots$$

Peebles (1980)

The scale *R* is not calculable from PT (we can only estimate it)

PT approach to LSS

Effects of short-scale fluctuations are encoded in counter-terms

Effective Field Theory approach to LSS

$$\partial_\tau \delta + \nabla[(1+\delta)\boldsymbol{v}] = 0$$

$$\partial_\tau \boldsymbol{v} + \mathcal{H}\boldsymbol{v} + \nabla\Phi + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = -c_s^2 \nabla\delta + \cdots$$

$$\nabla^2\Phi = \frac{3}{2}\mathcal{H}^2\Omega_m\delta$$

EFT operators

Baumann, Nicolis, Senatore, Zaldarriaga (2010)

Carrasco, Hertzberg, Senatore (2012)

$$P_{13}^{\rm UV}(k) = -\frac{61}{630\pi^2} P_{\rm lin}(k) k^2 \int_0^\infty dq P_{\rm lin}(q)$$

$$P_{\rm count.}(k) = -2c_s^2(\tau) k^2 P_{\rm lin}(k)$$

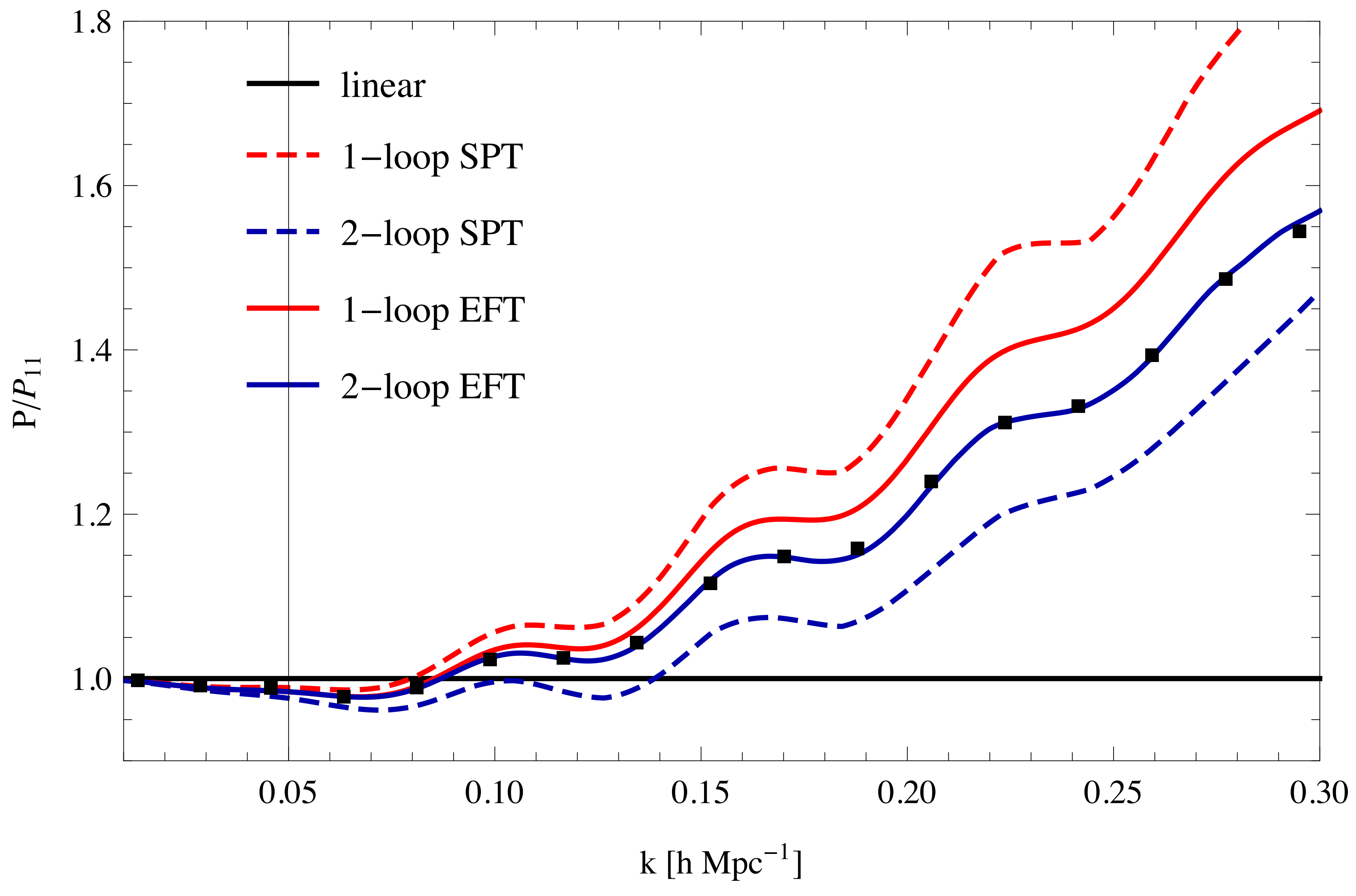
Leading UV sensitivity

PT approach to LSS

Including counter-terms (a single free parameter at two loops)

$$P_{\text{count.}}(k) \sim c_s^2(\tau)\left(2P_{13}^{q\to 0}(k) + 2P_{15}^{q\to 0}(k) + 2P_{24}^{q\to 0}(k) + P_{33-II}^{q\to 0}(k)\right)$$

Baldauf, Mercolli, Zaldarriaga (2015)



PT approach to LSS

Review:
Desjacques, Jeong, Schmidt (2016)

Bias expansion

$$\delta^{(g)} = \mathcal{F}[\nabla_i \nabla_j \Phi] = b_1 \delta + b_2 \delta^2 + b_{s^2} (\nabla_i \nabla_j \Phi)^2 + \tilde{b} \nabla^2 \delta \cdots$$

Galaxy formation is a local function of the tidal field + stochastic proceses

Write down all possible "operators" compatible with symmetries

Additional complication: non-locality in time…

PT approach to LSS

At one-loop in PT: 4 bias parameters for the power spectrum
(halos in real space)

The same bias parameters are in the bispectrum

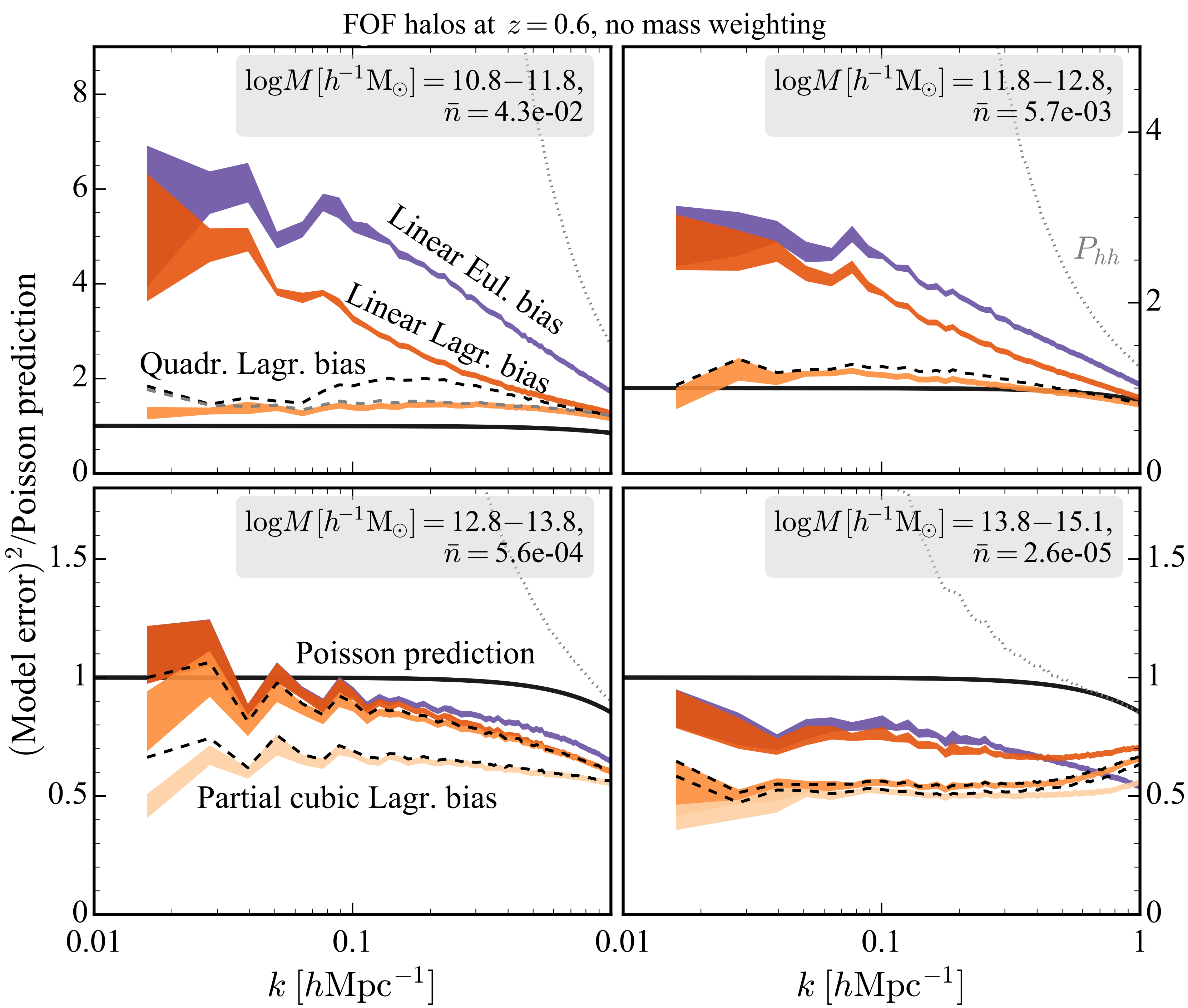
One-loop bias model valid up to k~0.2h/Mpc, with ~10% precision

For the 1% precision, we will have to do better…

Assassi, Schmittfull, MS, Zaldarriaga (in prep.)

PT approach to LSS

Do we really need all these parameters? Yes!



Preliminary!

PT approach to LSS

RSD and IR-resummation

Higher order statistics: bispectrum, trispectrum, covariance matrix…

Different flavors: Eulerian and Lagrangian EFT, TSPT,…

Fast methods for evaluation of loop integrals

Theoretical systematics and data analysis

Theoretical errors in PT

The goal is to increase k_{max}

Gravitational nonlinearities large

$$P_{\rm NL}(k) = P(k) + P_{\rm SPT}^{1-{\rm loop}}(k) + P_{\rm ct}(k) + \cdots$$

$$P_{\rm ct}(k) = -2R_p^2 k^2 P(k)$$

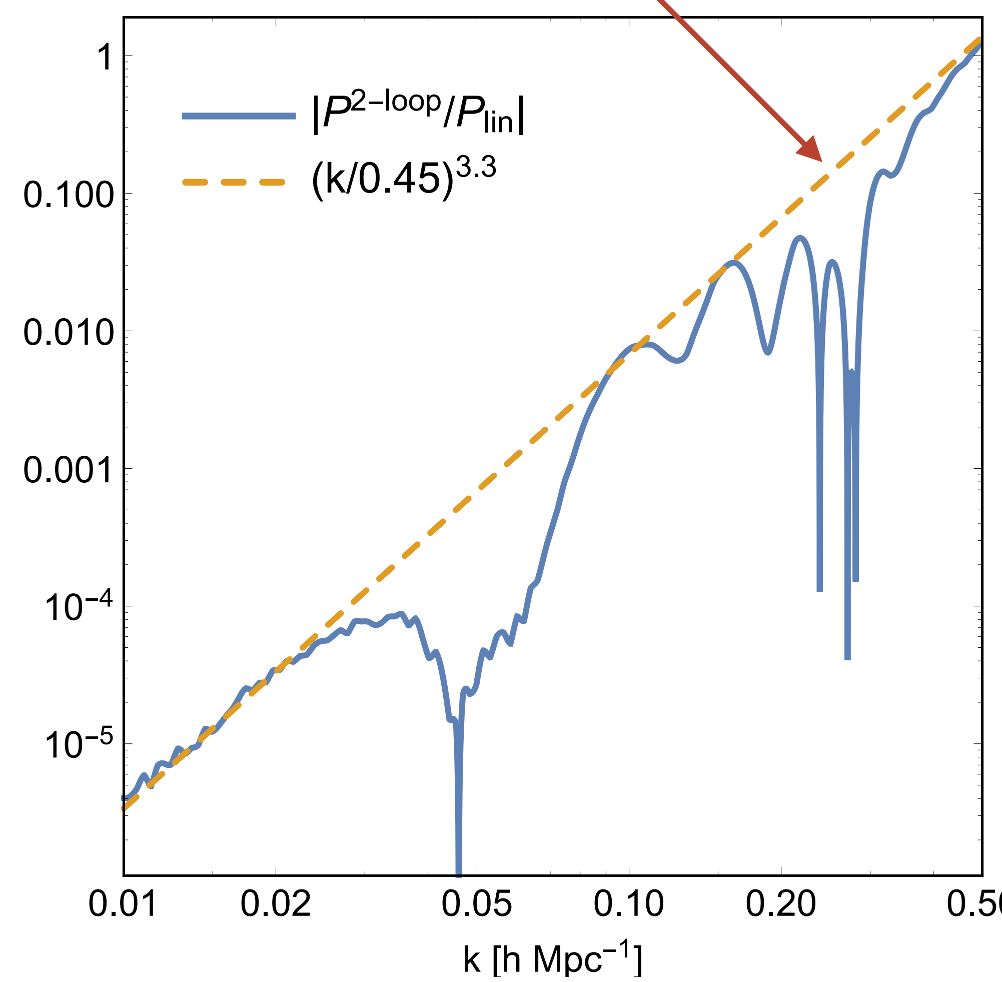
Higher order terms:
estimate of the error

$$P(k) \propto k^n$$

$$P^{l-{\rm loop}}(k)/P(k) \propto (k/k_{\rm NL})^{(3+n)l}$$

$$k_{\rm NL} \sim 0.3\ h{\rm Mpc}^{-1}\ , \quad n \sim -1.5$$

“error for 1-loop”



Theoretical errors in PT

The goal is to increase k_{max}

Gravitational nonlinearities large

$$P_{\rm NL}(k) = P(k) + P^{\rm 1-loop}_{\rm SPT}(k) + P_{\rm ct}(k) + \cdots$$

$$P_{\rm ct}(k) = -2R_p^2 k^2 P(k)$$

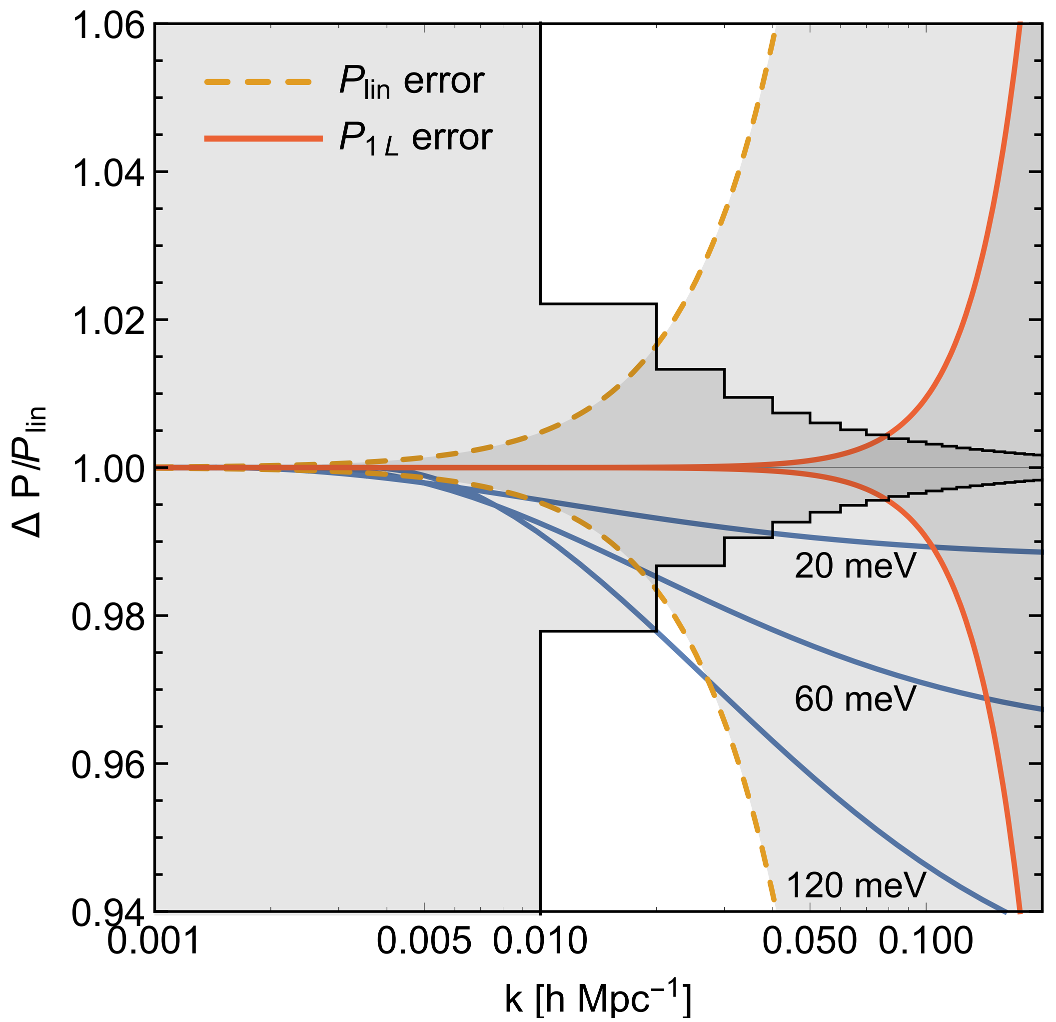
Higher order terms: estimate of the error

$$P(k) \propto k^n$$

$$P^{\rm l-loop}(k)/P(k) \propto (k/k_{\rm NL})^{(3+n)l}$$

$$k_{\rm NL} \sim 0.3\; h{\rm Mpc}^{-1}\;, \quad n \sim -1.5$$

These errors are large!



Theoretical errors in PT

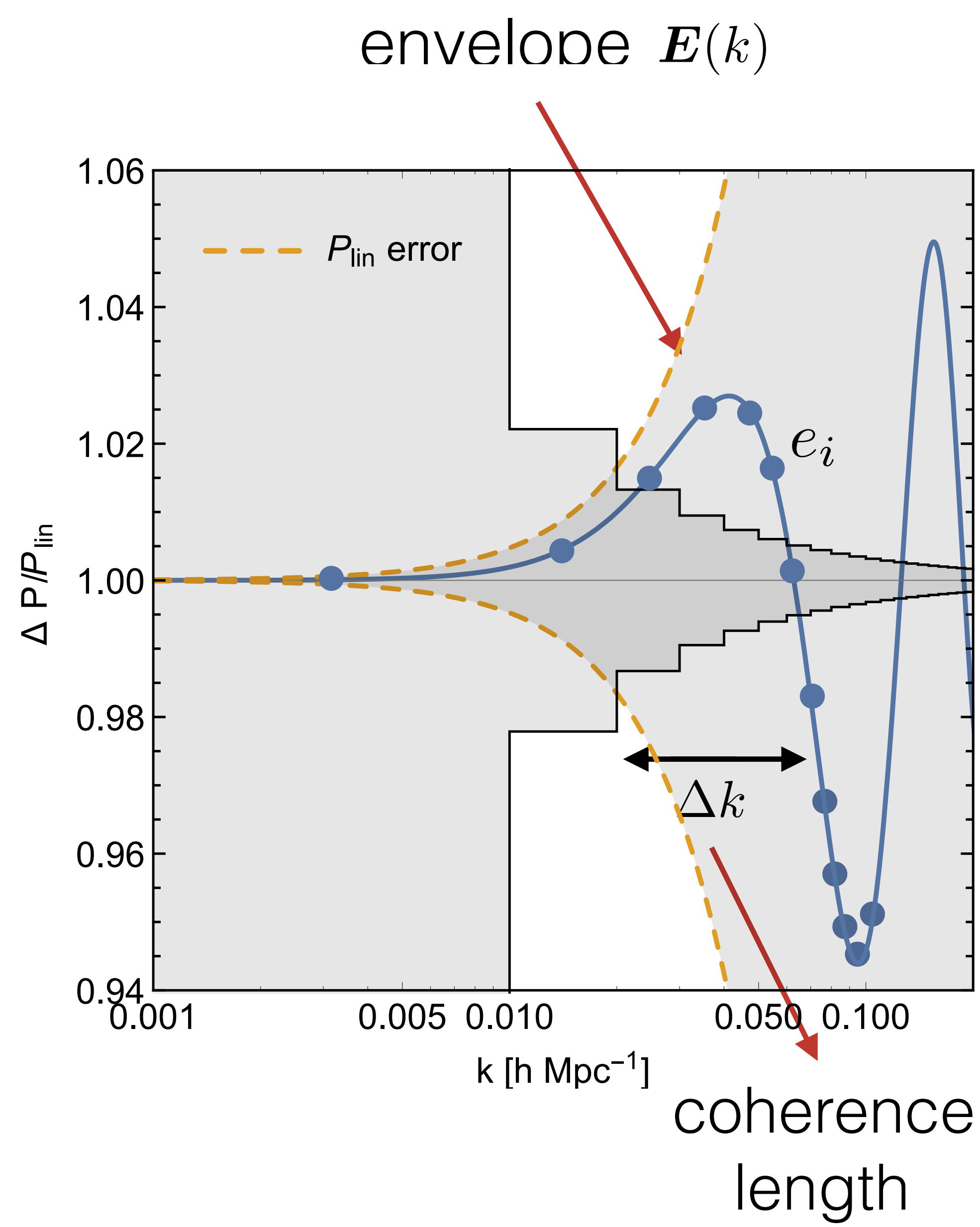
Marginalize over all possible models with given $\boldsymbol{E}(k)$ and Δk

$$\mathcal{L}_e = \frac{1}{\sqrt{(2\pi)^{N_c}|C_d|}} \exp\left[-\frac{1}{2}(\boldsymbol{d}-\boldsymbol{t}_{\rm f}-\boldsymbol{e})C_d^{-1}(\boldsymbol{d}-\boldsymbol{t}_{\rm f}-\boldsymbol{e})\right]$$
$$\times \frac{1}{\sqrt{(2\pi)^{N_c}|C_e|}} \exp\left[-\frac{1}{2}\boldsymbol{e}C_e^{-1}\boldsymbol{e}\right]$$

$$\mathcal{L} = \frac{1}{\sqrt{(2\pi)^{N_c}|C|}} \exp\left[-\frac{1}{2}(\boldsymbol{d}-\boldsymbol{t})C^{-1}(\boldsymbol{d}-\boldsymbol{t})\right]$$

$$C = C_d + C_e \qquad (C_e)_{ij} = E_i \rho_{ij} E_j$$

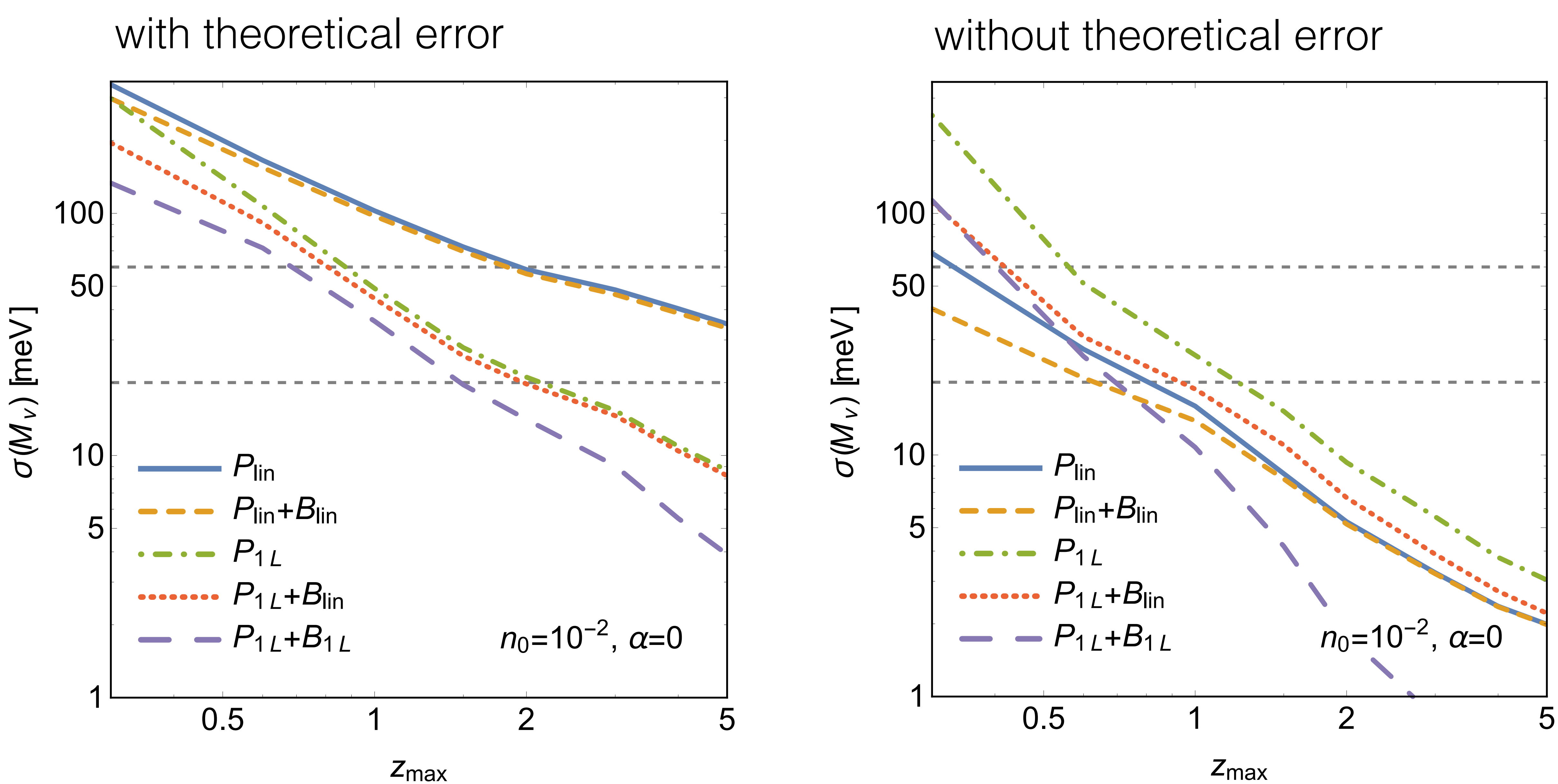
$$\rho_{ij} = \begin{cases} \exp\left[-(k_i-k_j)^2/2\Delta k^2\right] & P\,, \\ \prod_{\alpha=1}^{3} \exp\left[-(k_{i,\alpha}-k_{j,\alpha})^2/2\Delta k^2\right] & B\,. \end{cases}$$



A different proposal (without coherence length)
Audern, Lesgourgues, Bird, Haehnelt, Viel, JCAP 1301, 026 (2013)

Theoretical errors in PT

Marginalizing over all nuisance parameters



Part II

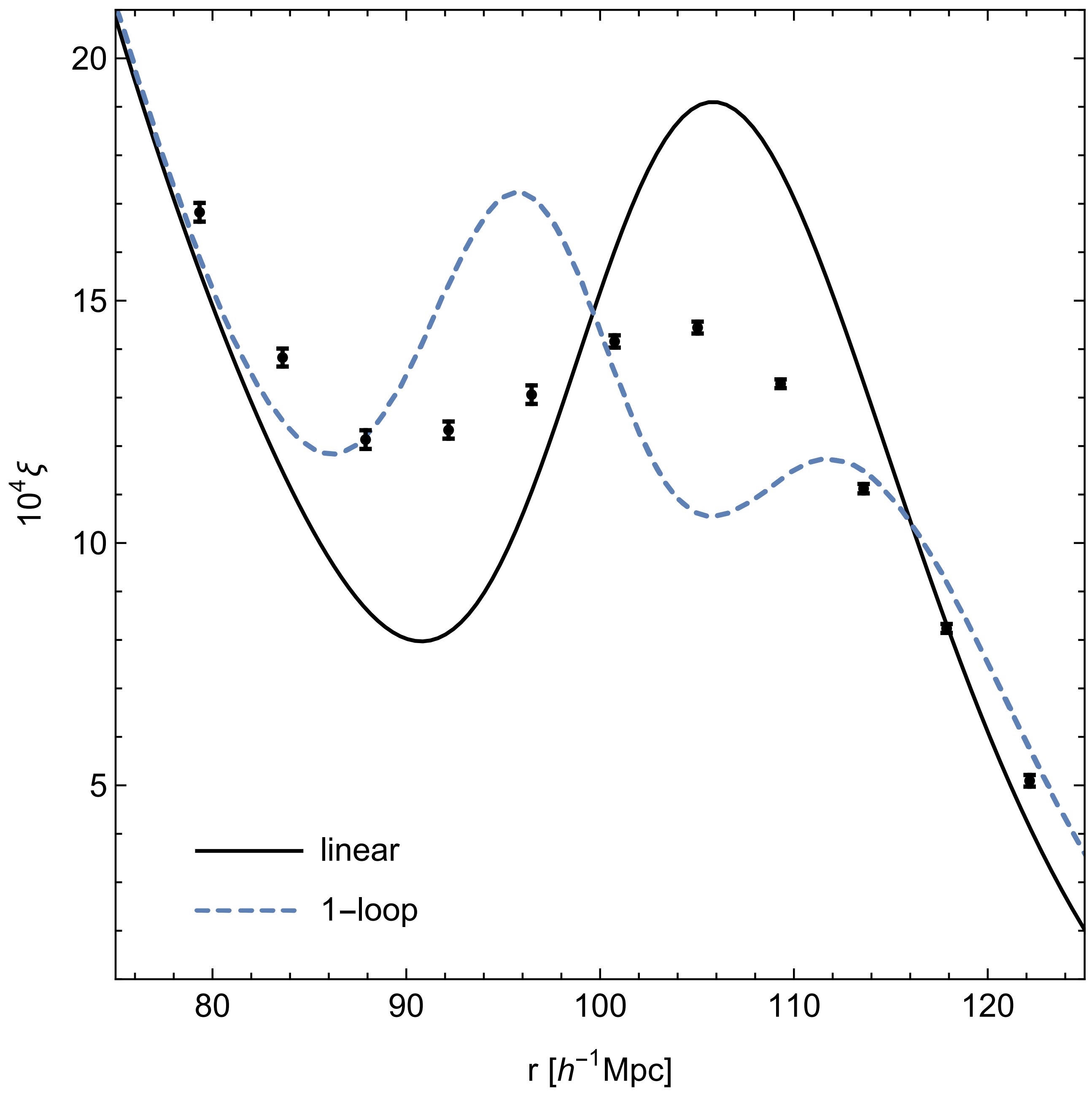
Modeling of the BAO peak

Modeling of the BAO peak

One well-known problem of the Eulerian PT

But PT should work at

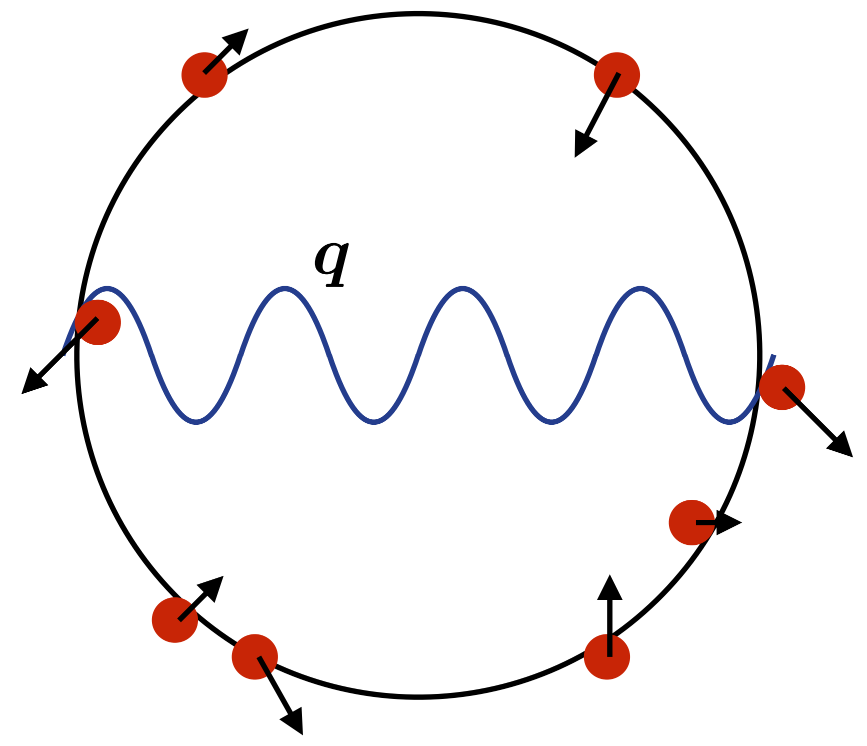
$\ell_{\rm BAO} \sim 100\ h^{-1}{\rm Mpc}$



Modeling of the BAO peak

Galaxies in free fall $\delta x^i \sim \nabla^i \phi \sim \frac{\nabla^i}{\nabla^2}\delta$

(not Zel'dovich displacements)



$q \ll 2\pi/\ell_{\rm BAO}$

no effect (exact squeezed limit)

$2\pi/\ell_{\rm BAO} < q \ll 2\pi/\sigma$

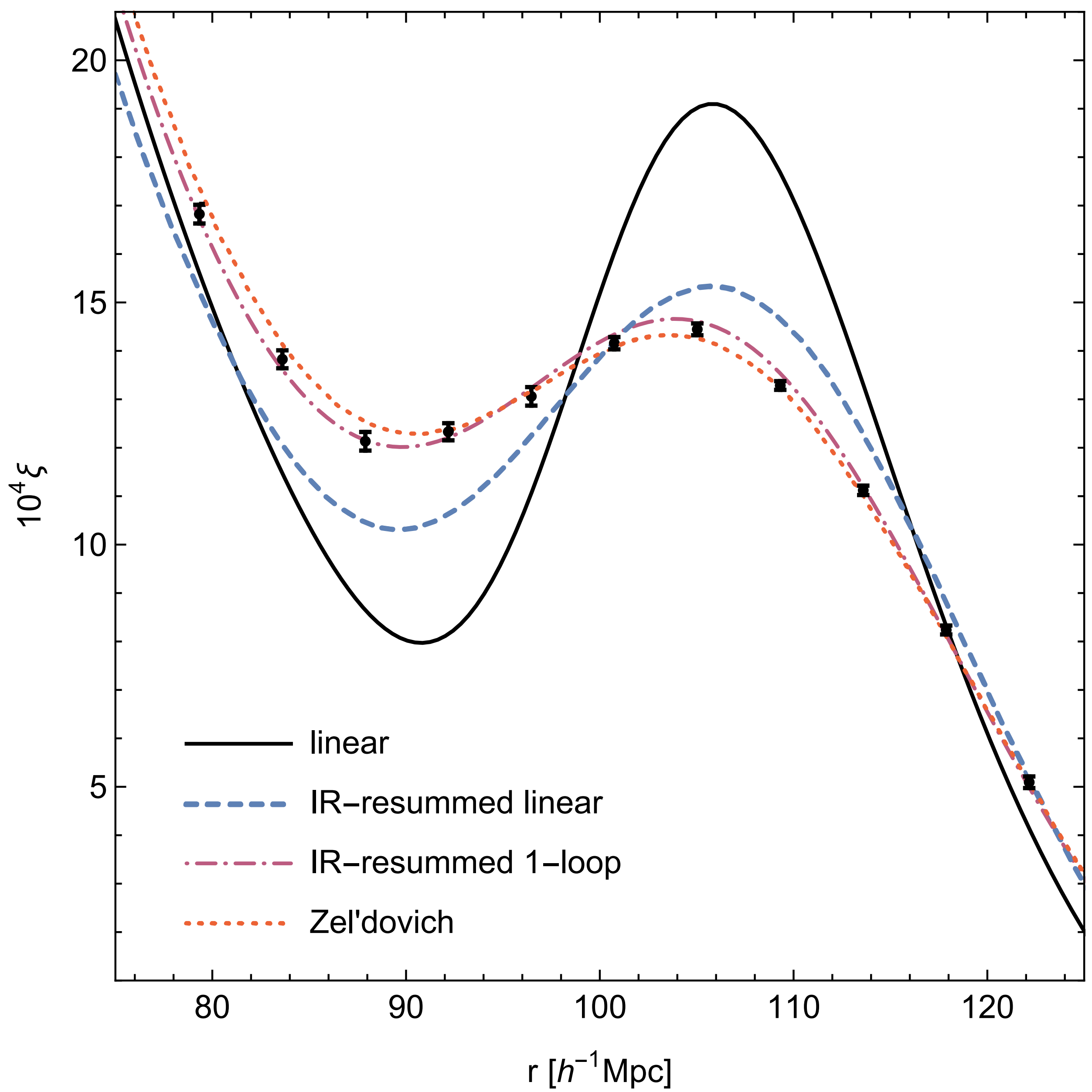
observable effect (spread of the peak)

Baldauf, Mirbabayi, MS, Zaldarriaga (2015)

$$\tilde{P}(k) = P^{nw}_{\rm lin}(k) + P^{nw}_{\rm 1-loop}(k)$$
$$+ e^{-\Sigma^2_{\epsilon k} k^2}(1 + \Sigma^2_{\epsilon k} k^2) P^w_{\rm lin}(k) + e^{-\Sigma^2_{\epsilon k} k^2} P^w_{\rm 1-loop}(k)$$

$$\Sigma^2_\Lambda \approx \frac{1}{6\pi^2}\int_0^\Lambda {\rm d}q P_{\rm lin}(q)[1 - j_0(q\ell_{\rm BAO}) + 2j_2(q\ell_{\rm BAO})]$$

Modeling of the BAO peak



Different form the standard formula for the spread of the BAO peak

Crocce, Scoccimarro (2007)

Eisenstein, Seo, White (2007)

Only long-short shifts

Senatore, Zaldarriaga (2014)

Baldauf, Mirbabayi, MS, Zaldarriaga (2015)

Vlah, Seljak, Chu, Feng (2015)

Blas, Garny, Ivanov, Sibiryakov (2016)

Senatore, Trevisan (2017)

Parameter-free modeling of the BAO peak (including bias, RSD…)

Modeling of the BAO peak

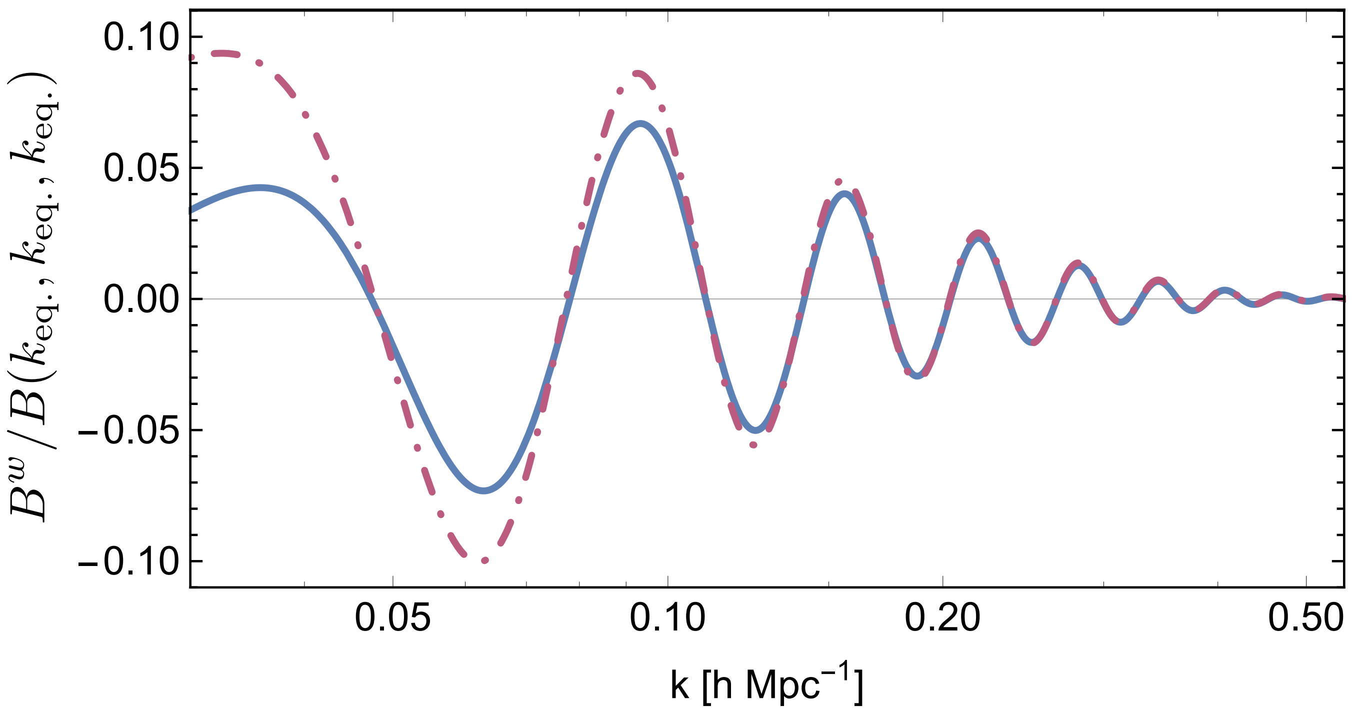
Clear connection to higher-point correlation functions

$$B_g^w \approx \frac{2\mu}{b_1}\frac{k}{q}\sin\left(\frac{x\mu}{2}\right) P_g(q)\cdot \ell_{\rm BAO}^{-1}\frac{d}{dk}P_g^w(k) \qquad \ell_{\rm BAO}^{-1} < q \ll k$$

$$x = q\ell_{\rm BAO}$$

$$\mu = \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}$$

Baldauf, Mirbabayi, MS, Zaldarriaga (2015)



Modeling of the BAO peak

The BAO peak is a playground for modified gravity theories

Possible test for different models that violate the EP

The infrared structure of correlators for arbitrary small *q*

Creminelli, Gleyzes, Hui, MS, Vernizzi (2013)

$$\langle \delta_{\vec{q}}\, \delta^{gA}_{\vec{k}_1} \delta^{gB}_{\vec{k}_2} \rangle'_{q\to 0} = -\lambda \frac{\vec{q}\cdot\vec{k}_1}{q^2} P_\delta(q) \langle \delta^{gA}_{\vec{k}_1} \delta^{gB}_{\vec{k}_2} \rangle'$$

Part III

Some open questions

Important questions for future LSS surveys

What is the optimal model for the galaxy power spectrum?

What do we gain using the bispectrum?

How to make a reliable estimate of theoretical errors?

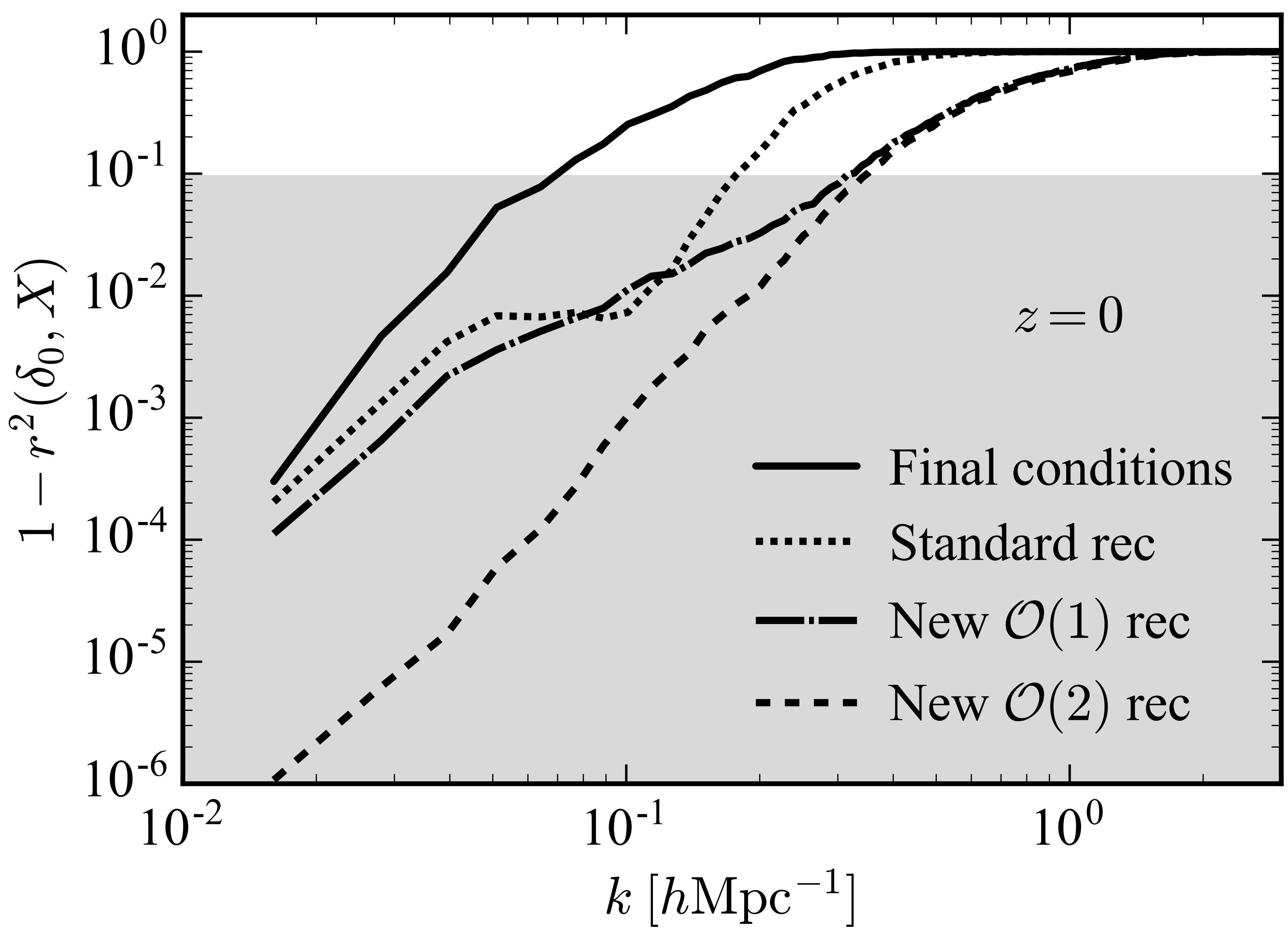
How to exploit “same-realization” measurements to improve theoretical models?

Should we think about alternatives, like reconstruction of the IC?

Reconstruction of the initial conditions

Can we do as well for haloes in redshift space?

Baldauf, Schmittfull, Zaldarriaga (2017)



Conclusions

Very good understanding of large-scale clustering of DM

More work needed for RSD and biased tracers

Much more work needed for higher order correlation functions

Alternatives to n-point functions promising but largely unexplored

Backup slides

Theoretical errors in PT

$$\boldsymbol{t} = (P_1, \ldots, P_n)$$
$$\boldsymbol{d} = (P_1^d, \ldots, P_n^d)$$

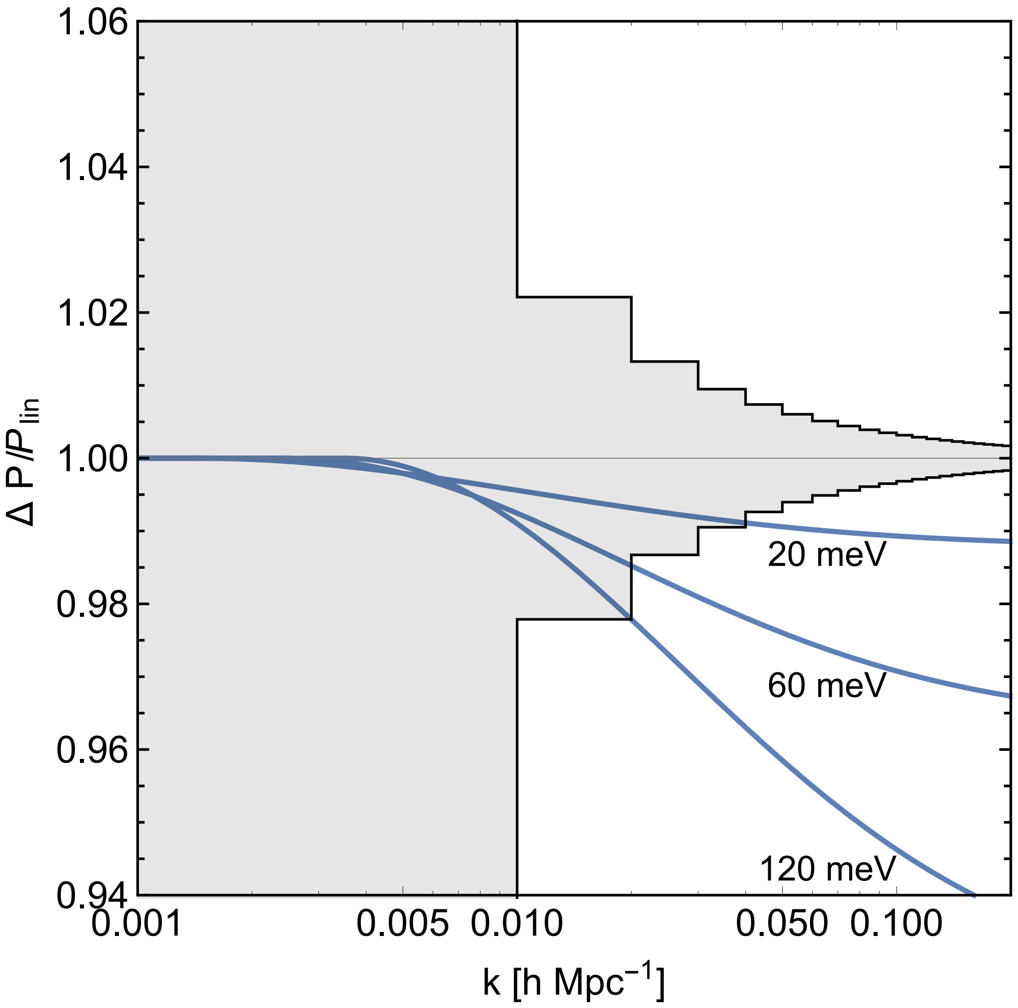
Likelihood:

$$\mathcal{L} = \frac{1}{\sqrt{(2\pi)^{N_c}|C_d|}} \exp\left[-\frac{1}{2}(\boldsymbol{d}-\boldsymbol{t})C_d^{-1}(\boldsymbol{d}-\boldsymbol{t})\right]$$

$$(C_d)_{ij} = (\Delta P_i)^2 \delta_{ij}$$

$$F_{ij} = -\left\langle \frac{\partial^2 \log \mathcal{L}}{\partial p_i \partial p_j} \right\rangle \Bigg|_{\boldsymbol{p}=\boldsymbol{p}_0} \qquad \boldsymbol{p} = (M_\nu, A)$$

$$\sigma(p_i) = \sqrt{(F^{-1})_{ii}}$$

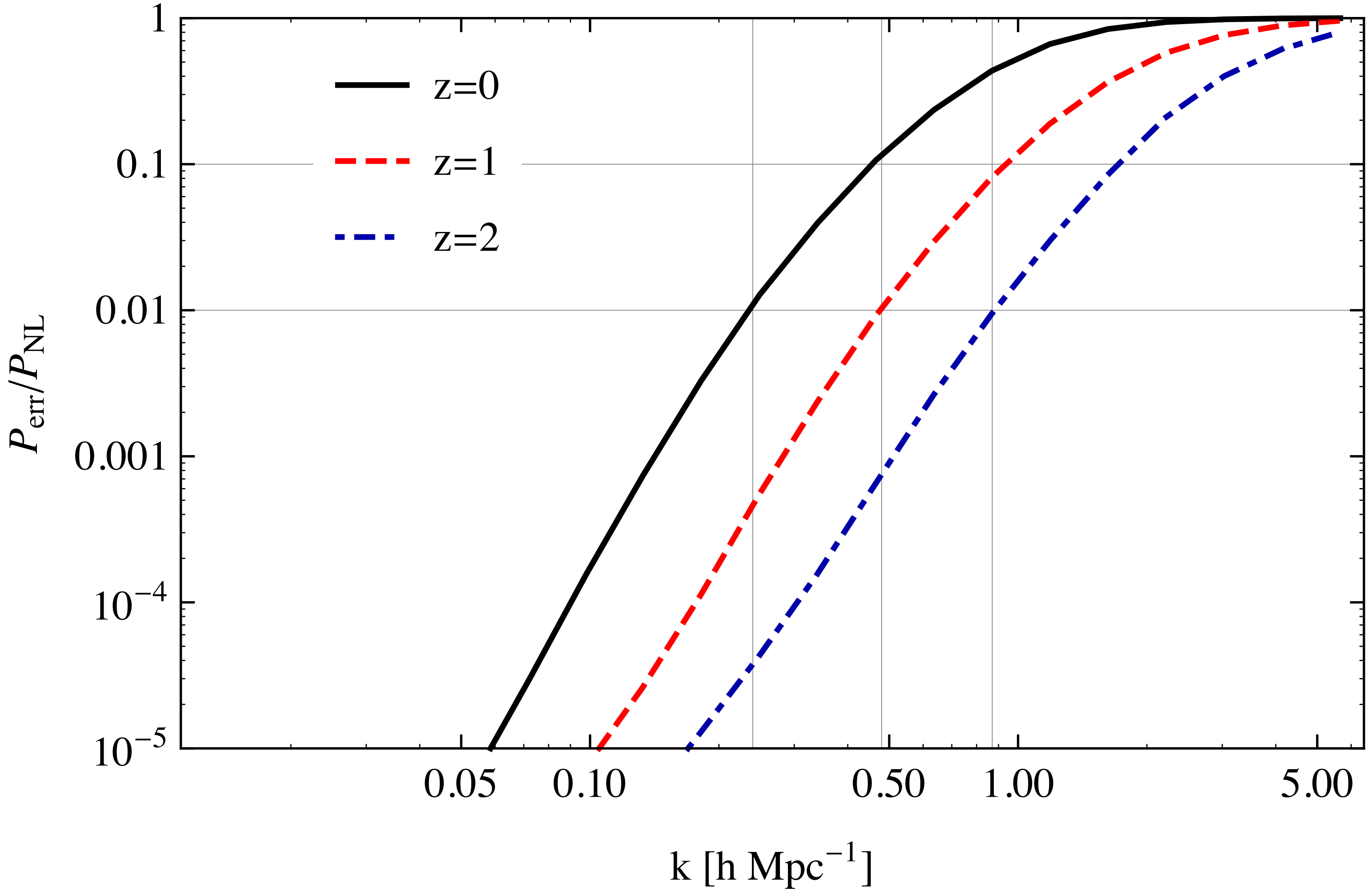


PT approach to LSS

Comparison on the level of the density field

$$1 - r^2 = 1 - \frac{\langle \delta_{\mathrm{PT}} \delta_{\mathrm{sim}} \rangle^2}{\langle \delta_{\mathrm{PT}} \delta_{\mathrm{PT}} \rangle \langle \delta_{\mathrm{sim}} \delta_{\mathrm{sim}} \rangle}$$

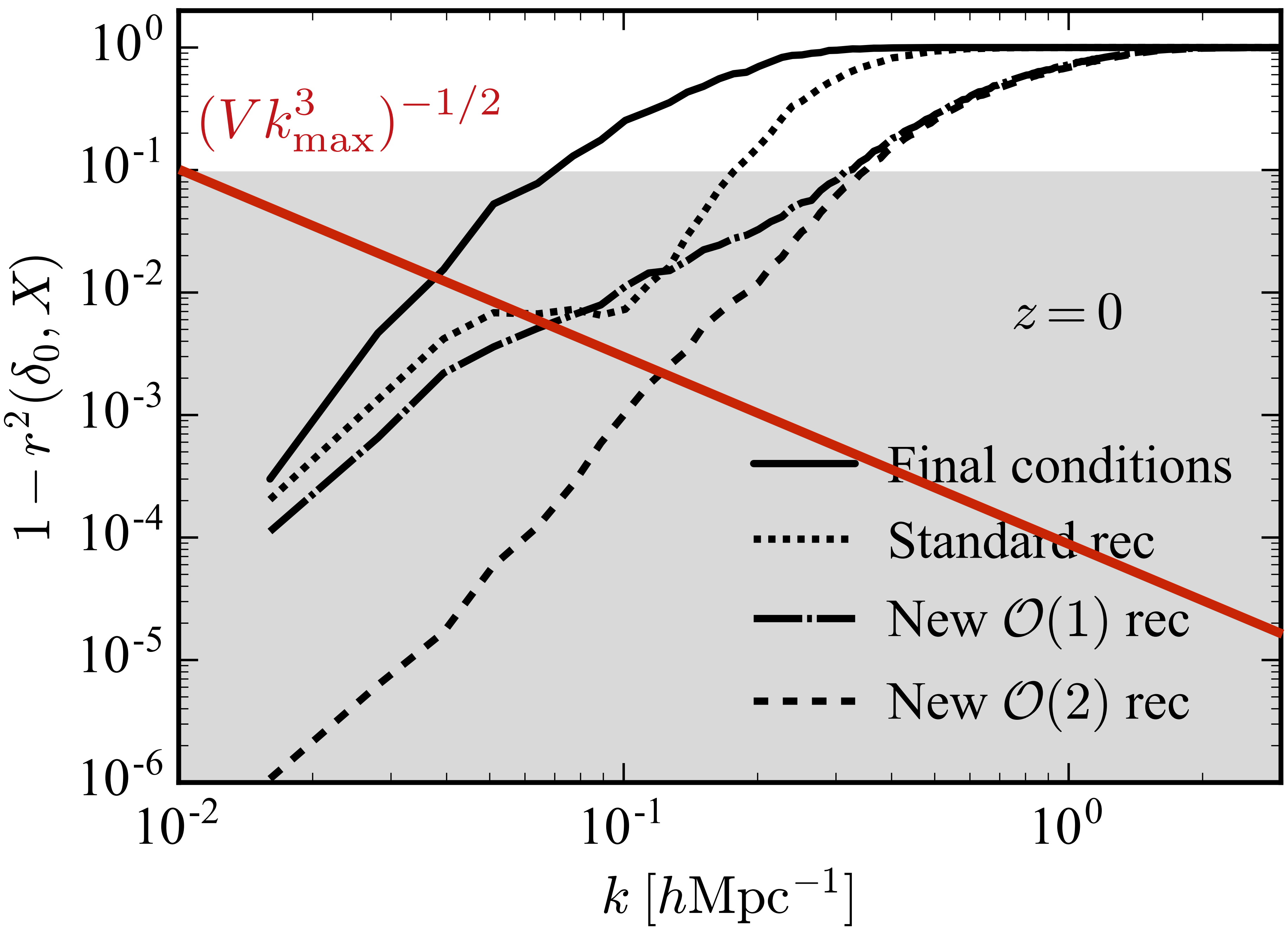
Baldauf, Schaan, Zaldarriaga (2015)



Theoretical uncertainties

Significant impact on data analysis!

Baldauf, Mirbabayi, MS, Zaldarriaga (2015)



Efficient evaluation of cosmological statistics

$$P_{22}(k) = 2\int_{\boldsymbol{q}} F_2^2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) P_{\rm lin}(q) P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|)$$

$$F_2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) = \frac{5}{14} + \frac{3k^2}{28q^2} + \frac{3k^2}{28|\boldsymbol{k}-\boldsymbol{q}|^2} - \frac{5q^2}{28|\boldsymbol{k}-\boldsymbol{q}|^2} - \frac{5|\boldsymbol{k}-\boldsymbol{q}|^2}{28q^2} + \frac{k^4}{14|\boldsymbol{k}-\boldsymbol{q}|^2 q^2}$$

A convolution integral — easy to solve using FFT(Log)

McEwen, Fang, Hirata, Blazek (2016) — FastPT

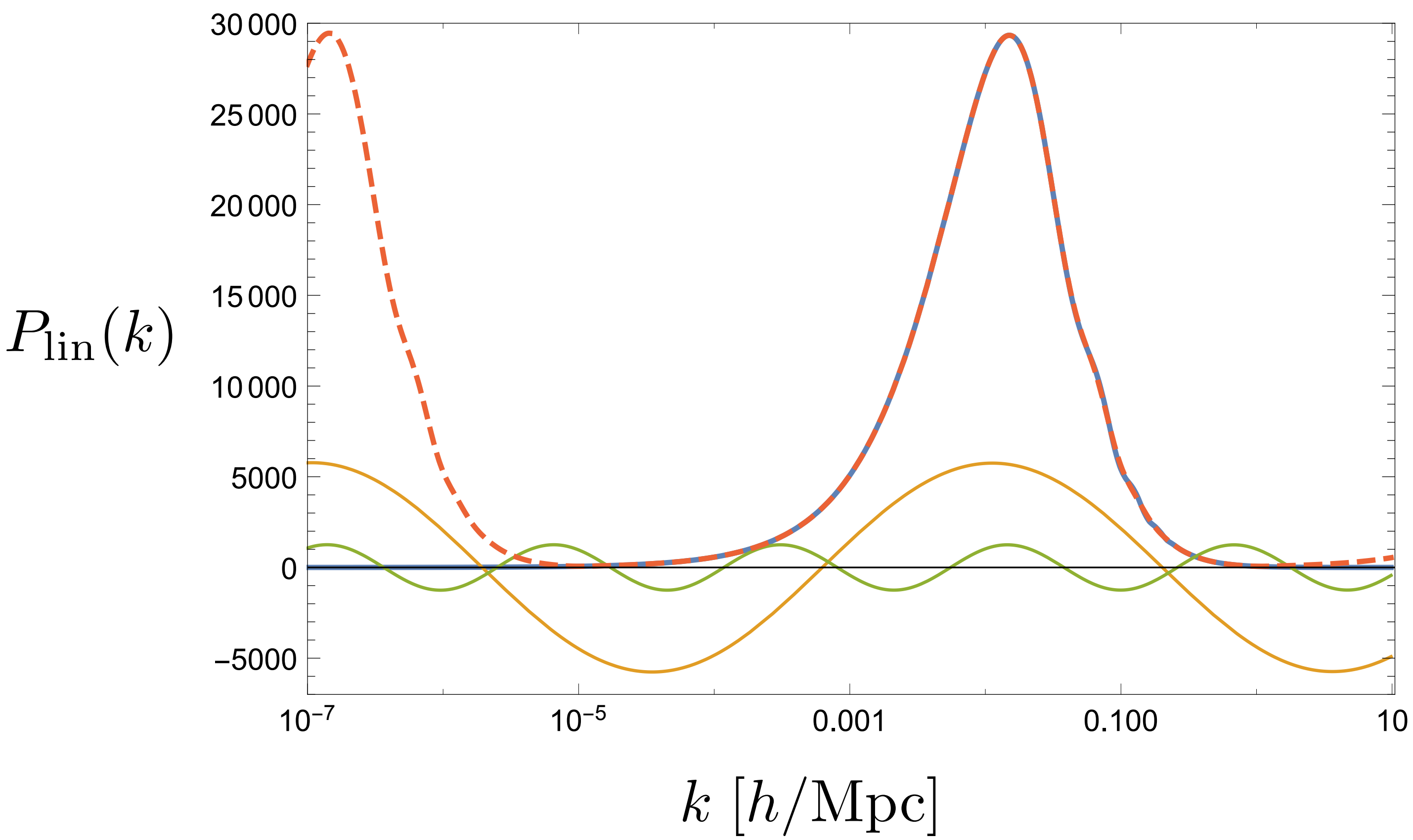
Schmittfull, Vlah, McDonald (2016)

This trick works only for the one-loop power spectrum

Efficient evaluation of cosmological statistics

A different point of view on FFTLog

Hamilton (2000)



$$\bar{P}_{\rm lin}(k_n) = \sum_{m=-N/2}^{m=N/2} c_m\, k_n^{\nu + i\eta_m}$$

Any cosmology can be written as a sum of power-law universes

All cosmology dependence is just in *c_m*

Efficient evaluation of cosmological statistics

Convolution integrals in PT

$$P_{22}(k) = 2\int_{\boldsymbol{q}} F_2^2(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) P_{\rm lin}(q) P_{\rm lin}(|\boldsymbol{k}-\boldsymbol{q}|)$$

MS, Baldauf, Zaldarriaga, Carrasco, Kollmeier (2017)

$$P_{22}(k) = k^3 \sum_{m_1, m_2} c_{m_1} k^{-2\nu_1} \cdot M_{22}(\nu_1, \nu_2) \cdot c_{m_2} k^{-2\nu_2}$$

$$M_{22}(\nu_1, \nu_2) = \frac{(\frac{3}{2}-\nu_{12})(\frac{1}{2}-\nu_{12})[\nu_1\nu_2(98\nu_{12}^2 - 14\nu_{12} + 36) - 91\nu_{12}^2 + 3\nu_{12} + 58]}{196\,\nu_1(1+\nu_1)(\frac{1}{2}-\nu_1)\,\nu_2(1+\nu_2)(\frac{1}{2}-\nu_2)}\, I(\nu_1, \nu_2).$$

$$\int_{\boldsymbol{q}} \frac{1}{q^{2\nu_1}|\boldsymbol{k}-\boldsymbol{q}|^{2\nu_2}} \equiv k^{3-2\nu_{12}}\, I(\nu_1, \nu_2)$$

$$I(\nu_1, \nu_2) = \frac{1}{8\pi^{3/2}} \frac{\Gamma(\frac{3}{2}-\nu_1)\Gamma(\frac{3}{2}-\nu_2)\Gamma(\nu_{12}-\frac{3}{2})}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(3-\nu_{12})}$$

Efficient evaluation of cosmological statistics

Angular power spectra and bispectra

$$C_\ell^{(g)} = \frac{2}{\pi} b_1^2 \int_0^\infty \mathrm{d}\chi \int_0^\infty \mathrm{d}\chi' \, \mathsf{W}_g^{(1)}(\chi) \mathsf{W}_g^{(1)}(\chi') \boxed{\int_0^\infty \frac{\mathrm{d}k}{k} j_\ell(k\chi) j_\ell(k\chi') \left[k^3 P_{in}(k)\right]}$$

Numerically challenging

Using FFTLog

Assassi, MS, Zaldarriaga (2017)

Gebhardt, Jeong (2017)

$$\int_0^\infty \frac{\mathrm{d}k}{k} j_\ell(k\chi) j_\ell(k\chi') [k^3 P_{in}(k)] = \sum_m c_m \int_0^\infty \frac{\mathrm{d}k}{k} j_\ell(k\chi) j_\ell(k\chi') k^{\nu_m}$$

$$\mathsf{I}_\ell(\nu, t) \equiv 4\pi \int_0^\infty \mathrm{d}v \; v^{\nu-1} j_\ell(v) j_\ell(vt) = \frac{2^{\nu-1}\pi^2 \, \Gamma(\ell + \frac{\nu}{2})}{\Gamma(\frac{3-\nu}{2}) \Gamma(\ell + \frac{3}{2})} \, t^\ell \; {}_2F_1\left(\frac{\nu-1}{2}, \ell + \frac{\nu}{2}, \ell + \frac{3}{2}, t^2\right)$$

Bispectrum for CMB primary anisotropies, lensing, galaxies…